Green Asset Pricing*

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Abstract

This paper demonstrates that empirically grounding the discount factor significantly influences the determination of the carbon price. Using two complementary nonlinear statistical approaches, we assess which utility formulations and corresponding stochastic discount factors best align with U.S. data. We provide evidence that habit formation is essential for capturing the time variation in the stochastic discount factor necessary to match the data. This increased time variation raises the carbon price by 32% and makes it five times more procyclical compared to standard models. The heightened procyclicality reduces aggregate risk, the risk premium, and the need for precautionary savings.

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1 Introduction

In this paper, we explore how empirically grounding discount rates shapes optimal carbon pricing. At its core, determining the social cost of carbon (SCC) is fundamentally an asset pricing problem. The SCC represents the net present value of the marginal damages caused by an additional unit of carbon dioxide emitted today. Central to this calculation are the utility function and the stochastic discount factor (SDF), which serve to translate the future economic damages of carbon emissions into present values.[1] This paper contends that discount rates used in current climate models often lack empirical grounding. Without realistic discounting, the carbon price could be systematically underestimated and would fail to adjust adequately during economic recessions.

Determining the appropriate price of carbon remains a contentious issue. This is highlighted by the debate between William Nordhaus and Nicholas Stern on the proper discount rate for future climate-related damages. Most studies base their recommendations on deterministic versions of the discount factor (e.g., Nordhaus, 1991, 2008, 2017). Yet, both the stochastic nature of the discount factor and its consistency with asset price data have been largely overlooked in quantitative assessments of the SCC. Recent advances in modern macro-finance theory present a promising solution to the climate discount challenge.

An emerging literature on climate risk has introduced the early resolution of uncertainty into carbon pricing (e.g., Cai and Lontzek, 2019; Bansal, Kiku, and Ochoa, 2019; Barnett, Brock, Hansen, and Hong, 2020; Van Den Bremer and Van Der Ploeg, 2021; Traeger, 2021). This body of work emphasizes that uncertainty significantly influences the SCC, through the mechanism of risk preferences affecting the SDF. However, these studies often focus—at best—on the mean of the risk-free rate as the primary moment for plausible discounting. Other considerations, such as the volatility of the risk free rate and risk premium data are not considered while relevant for guiding discounting. Moreover, these studies do not consider a formal statistical metric for assessing the accuracy and performance of models in matching macro-financial aggregates and asset prices.

Our study differentiates itself by empirically justifying our choice of discount factors through matching key asset pricing data. Leveraging recent advances in asset pricing theory, the main goal of this paper is to evaluate how finance, particularly through em-

[1] Climate change represents a significant market failure, as it reflects the inability of markets to internalize the cost of carbon emissions. Setting a carbon tax equal to the SCC is considered the most effective strategy to correct this failure and reduce emissions. The SCC has become a critical metric in policy design, as illustrated by the U.S. administration's adoption of a $51 per ton CO_2 rate in 2020.

pirically grounded discounting, affects carbon pricing. To match asset pricing moments, we develop a framework for the U.S. economy that integrates core elements of finance, such as risk preferences and long-run risk (Bansal and Yaron, 2004; Kung and Schmid, 2015). To capture realistic fluctuations in the SDF (e.g., Cochrane and Hansen, 1992), we also introduce a specification of slow-moving internal habits. Compared to existing literature, our framework emphasizes time variation in the SDF as an important determinant of optimal carbon pricing. Additionally, we incorporate climate-related components from the integrated assessment model literature (e.g. Golosov, Hassler, Krusell, and Tsyvinski, 2014; Nordhaus, 2017), including carbon accumulation, damages, and abatement technologies.

Our framework introduces novel elements that advance the current state of the literature. First, our economy comprises an endogenous slope of consumption growth (e.g., Kung and Schmid, 2015) that is affected by climate damages, thereby creating long-run climate risk (Bansal et al., 2019; Casey, Fried, and Gibson, 2024; Nath, Ramey, and Klenow, 2024). Additionally, the social cost of carbon is driven not only by climate damages on production, but also by households' utility functions. We define the latter as depending on both consumption and environmental quality; as the stock of emissions harms consumers, utility is modeled as a function of the ratio between consumption and accumulated atmospheric emissions.[2]

Beyond these core elements, our approach remains a priori agnostic regarding the specific utility function that can be used to jointly achieve a 6 percent risk premium and a 0.4 percent risk-free rate. Traditional macro-finance models typically rely on either long-run risk or consumption habits, often in isolation, without a statistical comparison between each preference formulation. This paper contends that the choice of utility function is nontrivial for the purposes of carbon pricing and proposes two complementary statistical methods for carrying out relevant analyses.[3] First, we use the simulated method of moments (SMM) (Hansen and Singleton, 1982) to evaluate the model's performance in matching key asset pricing moments related to the risk-free rate and equity premium. As is common practice in macro-finance (e.g., Mehra and Prescott, 1985; Jer-

[2]This preference for carbon has been empirically documented such as in Baker, Bergstresser, Serafeim, and Wurgler (2018) and Zerbib (2019), have shown that pro-environmental preferences affect asset pricing dynamics. They both find a positive and significant premium between green and nongreen bonds (the 'greenium'), suggesting an important role for these preferences in the ongoing debate on carbon taxation. In Pàstor, Stambaugh, and Taylor (2021), the effect of climate change on financial returns is explained by introducing a green factor that captures environmental concerns among investors.

[3]Our quantitative and statistical assessment employs third-order perturbations in order to capture time variation in risk premia and uncertainty.

mann, 1998), this economy is subject to a single source of uncertainty in productivity. We find our model to be capable of addressing long-standing asset pricing anomalies, contrasting sharply with the standard macro-environmental approach by generating a realistic equity premium and risk-free rate while remaining consistent with macroeconomic aggregate dynamics. In a framework consistent with asset price data, our results indicate that the carbon price is much higher and more procyclical than previously reported in the climate finance literature. This divergence is attributed to consumption habits, which induce important time variation in the SDF to match both the observed volatility in the risk-free rate and the predictability of asset returns.

To assess whether consumption habits should be considered a key component of the climate risk literature, our second statistical method employs Bayesian estimation techniques (e.g., Smets and Wouters, 2007). To allow the estimation of the structural model, fluctuations are driven by various sources of uncertainty.[4] Using U.S. data from 1973 to 2023 on GDP, consumption, CO_2 emissions, equity premiums, and the risk-free rate, we evaluate four nested versions of the utility function, ranging from the most advanced—with both habits and early resolution of uncertainty—to a simple power utility. By imposing each utility specification as a data-generating process, Bayesian techniques allow for statistical evaluation and counterfactual analysis, simulating the U.S. economy's path under an optimal carbon price policy.

From this two-step empirical methodology, we derive three main results. A primary finding is the importance of matching the risk premium and risk-free rate in statistically determining the most suitable preference formulation, and thus the discount rate. Our estimation techniques show that the best predictive performance is achieved in a model incorporating both consumption habits and recursive utility. While it is well known from work by Bansal and Yaron (2004) that recursive utility struggles to generate enough volatility in the risk-free rate, adding habits introduces sufficient fluctuations in the SDF. This improves the model's predictions for the risk-free rate and enhances asset return predictability

A second significant finding is that consumption habits strongly influence the SDF and, consequently, both the level of carbon price and its dynamics. Habit formation amplifies the precautionary savings motive, which raises the SDF. A higher SDF places greater weight on future climate damages, resulting in a social cost of carbon (SCC) that

[4]Widespread in macroeconomics, the estimation requires at least as many structural shocks as observable variables. Macro-finance also features alternative sources of fluctuations (e.g., Bansal and Yaron, 2004; Basu and Bundick, 2017).

is 32% higher than in standard models that do not account for asset price data. Matching asset pricing data not only affects the level but also the time variation of the carbon price, making it five times more procyclical than in traditional approaches.[5] The heightened time variation in the SDF under consumption habits strongly exacerbates procylicality, making the carbon price five times more responsive to economic fluctuations. The influence of consumption habits on the SDF, as discussed by Abel (1999) and Campbell and Cochrane (1999), increases agents' risk-aversion when current consumption falls below past levels. This heightened risk aversion increases the volatility of marginal utility. Consequently, during recessions, the SDF declines more sharply, intensifying the drop in the carbon price.

A third and final result concerns the link between the optimal carbon price and the SDF, challenging the traditional dichotomy between macroeconomics and finance (e.g., Cochrane, 2017). In a framework consistent with asset price data, a strongly procyclical carbon price significantly dampens economic fluctuations. The carbon price makes production relatively cheaper (more expensive) during recessions (expansions), which helps to reduce macroeconomic volatility. By lowering aggregate risk, the carbon price reduces the welfare cost of economic fluctuations.[6] In a more stable economy, investors require less consumption smoothing and demand lower compensation for holding risky assets. The asset pricing implications of carbon taxing are significant, with a 1 percent decrease in the risk premium and a 1 percent increase in the risk-free rate relative to the laissez-faire economy.

An important contribution of our paper is its estimation of the optimal environmental tax using Bayesian methods. From a computational perspective, the challenge lies in developing a tractable nonlinear estimation method, as uncertainty—and, consequently, higher-order terms in the Taylor expansion—plays a central role in our analysis. The literature typically employs weak information techniques, wherein a model is evaluated based on a few selected moments. Our approach, however, imposes the structural model as the data-generating process that replicates the full realization of observable variables. Consequently, the estimation does not rely on an arbitrary set of moments but instead on the exact replication of the sample. To our knowledge, our paper is the first to estimate

[5] The concept of a procyclical carbon price is not new in deterministic and climate risk models. Since emissions rise with output, the carbon tax naturally increases (decreases) during expansions (recessions), as noted by Heutel (2012) using a power utility. We show that early resolution of uncertainty, commonly used in climate risk models, produces even lower procyclicality than the power utility framework.

[6] As shown by Tallarini (2000), the welfare cost of business cycle fluctuations is considerably higher in models that generate realistic risk premiums.

a macro-finance model with an environmental externality using full-information techniques. A key advantage of our approach is that it allows us to estimate the laissez-faire equilibrium using U.S. data and then provide a counterfactual scenario under alternative policies. Our methods could be applied to introduce risk in other policy contexts, such as fiscal and monetary policies.

Our work is connected to a strand of the literature in which risk does not independently influence the determination of the carbon price. This line of research implements optimal carbon taxes with power utility under certainty equivalence. For example, Nordhaus (1991, 2008, 2017) provides periodic assessments of the SCC under an exogenously growing economy with abatement curves. In contrast, Golosov et al. (2014) derive the optimal carbon tax in a multisector neoclassical model with fossil fuels. All of these studies uses power utility in a deterministic setting and show that discount rate calibration significantly impacts the SCC. In contrast, Heutel (2012) produced one of the first studies to consider the SCC from a business-cycle perspective with stochastic productivity. Although this is a linearized model under certainty equivalence, Heutel (2012) finds that the optimal carbon tax is relatively procyclical. Our paper contributes to these various approaches by introducing risk and providing an empirical foundation for the determination of the SCC. In doing so, we are able to document how the stochastic nature of the discount factor matters in the determination of the carbon price.

A second strand of the literature on climate risk rigorously examines the impact of risk and uncertainty on the determination of the carbon price.[7] The studies that comprise this line of research all share the common assumption of early resolution of uncertainty to feature realistic discounting, but they differ regarding their modelling assumptions. For instance, Bansal et al. (2019) explore the asset pricing implications of climate change as a source of long-run risk. Another approach is that of Van Der Ploeg, Hambel, and Kraft (2020), who derived the optimal risky carbon tax in an endogenous-growth model with both dirty and green capital. Van Den Bremer and Van Der Ploeg (2021) study the effect of risk attitudes and uncertainty on the social cost of carbon, with closed-form expressions. Similarly, Cai and Lontzek (2019); Traeger (2021); Folini, Friedl, Kübler, and Scheidegger (2024) show how uncertainty impacts the level of the social cost of carbon in an extended version of DICE with many sources of uncertainty. Finally, another major concern for policymakers is that the predicted effect of climate policies on the economy is

[7] An extended review of the literature on climate-related financial economics, which studies interactions between climate change and financial markets, can be found in work by Giglio, Kelly, and Stroebel (2020). Similarly, a review of the macrofinancial implications of climate change is provided by Van Der Ploeg (2020)

strongly model-dependent. This issue is studied by Barnett, Brock, and Hansen (2021), who demonstrate how model uncertainty can influence policy formulation. A common feature of this climate risk literature is the widespread use of recursive preferences and matching the mean of the risk-free rate as the primary guide for discounting. Conversely, this paper provides an assessment of the degree to which time variation in the SDF is also important for matching the *volatility* of the risk-free rate.

2 The Model

Following Kung and Schmid (2015), we introduce endogenous growth into a version of the neoclassical growth model featuring a non-standard preference specification. As initially demonstrated by King, Plosser, and Rebelo (1988), balanced growth can be obtained in the neoclassical growth model by introducing labor-augmenting technological progress. As opposed to the real business cycle literature, in which the economy's growth rate is exogenously determined, here we endogenize the rate of labor-augmenting technological progress by allowing firms to invest in research and development (R&D).[8] Given that our main focus is the pricing of the environmental externality, we have kept the endogenous growth component of the model as simple as possible.

Similarly to Kung and Schmid (2015), we describe a growing economy in which some variables exhibit a time trend. As is usual practice, we then solve a version of the model in which non-stationary variables are detrended.[9] To clarify how endogenous growth affects the model's dynamics, we subsequently derive some key model implications using the stationary equilibrium. Variables that exhibit a time trend are growing in the steady state, and we use capital letters to denote this set of variables. Stationary variables, which display no time trend, are denoted using lowercase letters.

2.1 Firms

The representative firm produces the economy's final output good Y_t using labor n_{Yt} and capital K_{Yt} as production inputs.[10] The production function, which has a Cobb-

[8]Imposing restrictions on preferences is also necessary to derive an equilibrium consistent with balanced growth.

[9]Deriving a stationary equilibrium that is consistent with balanced growth requires several assumptions, which we only briefly discuss in the main text. A detailed analysis is presented in section A of the online appendix.

[10]Following King et al. (1988), since time devoted to work-related activities is finite, hours worked cannot increase in the steady state. In the non-stationary economy, therefore, the growth rate of n_{Yt} is

Douglas form, is given as

$$Y_t = \exp(-\zeta x_t) A \varepsilon_{At} K_{Yt}^{\alpha} (n_{Yt} H_t)^{1-\alpha}, \tag{1}$$

where A is the fixed level of productivity and ε_{At} is the exogenous technology shock governed by the process

$$\log \varepsilon_{At} = \rho_A \log \varepsilon_{At-1} + \eta_{At}, \tag{2}$$

where η_{At} is normally distributed with mean zero and standard deviation $\text{std}(\eta_{At})$.[11]

The term $\exp(-\zeta x_t)$ represents the climate damage to production, an emissions-driven reduction in productivity, that is commonly used in the IAM literature (e.g., Nordhaus, 1991). In line with Golosov et al. (2014), the damage takes the form of an exponential function, where ζ is a parameter that determines the magnitude of the damage. Notice that the stock of emissions, x_t, is assumed to be stationary. The rationale for this assumption is discussed in Section 2.2.

Labor-augmenting technological progress takes the form of human capital accumulation, the stock of which is denoted by H_t. Following Kung and Schmid (2015), our aim is to account for the fact that R&D investment is a key decision with significant long-run implications, as it impacts the economy's growth rate. The accumulation of human capital is governed by the following law of motion:

$$H_{t+1} = H_t + I_{Ht}. \tag{3}$$

In this expression, I_{Ht} is the amount invested in human capital, which determines the evolution of labor-augmenting technical progress H_t.

Investment in technological progress depends not only on the quantity of capital K_{Ht} and labor n_{Ht} allocated to RD, but also on the level of human capital H_t:

$$I_{Ht} = \exp(-\zeta x_t) \kappa \varepsilon_{At} K_{Ht}^{\xi} (n_{Ht} H_t)^{1-\xi}. \tag{4}$$

In this equation, the parameter κ represents the efficiency of the economy's R&D process by determining the increase in investment that can be achieved for a given amount of resources that firms devote to R&D. The parameter ξ denotes the capital intensity of the R&D sector, while n_{Ht} denotes the number of hours worked in the R&D sector.

zero.

[11]With labor-augmenting technological progress, balanced growth requires the exogenous technology shock process ε_{At} to be stationary (e.g., King et al., 1988).

In light of the evidence documented by Casey et al. (2024), the damage function $\exp(-\zeta x_t)$ also impacts the production function of R&D investment. Thus, in line with the latest available evidence, our model accounts for the fact that the environmental externality can be a source of long-run risk on consumption growth (e.g., Bansal and Yaron, 2004; Bansal et al., 2019).

The economy's capacity to convert R&D expenditures into long-term growth not only depends on the economy's stock of human capital H_t, but also on exogenous variations in technological progress ε_{At}. For the sake of parsimony, the TFP shock is common across sectors.[12] Following Jermann (1998), capital accumulation is subject to adjustment costs. Unlike in Jermann's seminal contribution, however, firms in our endogenous growth economy also need to decide how to allocate the aggregate capital stock, K_t, between the production of the final good and R&D:

$$K_t = K_{Yt} + K_{Ht}. \tag{5}$$

The economy's capital stock is governed by the following law of motion:

$$K_{t+1} = (1-\delta_K)K_t + \left(\frac{\chi_1}{1-\epsilon}\left(\frac{I_{Kt}}{K_t}\right)^{1-\epsilon} + \chi_2\right)K_t, \tag{6}$$

where I_{Kt} is the amount of resources devoted to physical capital accumulation. The stock of physical capital depreciates at the rate δ_K, while χ_1 and χ_2 are parameters that are chosen so that the economy's deterministic steady state is independent of adjustment costs.[13] The parameter ϵ controls the elasticity of the supply of capital with respect to investment. When set to zero, the supply of capital is perfectly elastic and the economy reduces to a model without adjustment costs.

The firm's objective is to maximize shareholder value by solving the following optimization problem:

$$\max_{Y_t, K_t, K_{Yt}, K_{Ht}, n_{Yt}, I_{Ht}, \mu_t, e_t} E_0 \sum_{t=0}^{\infty} m_t D_t \tag{7}$$

where the firm's dividends are given as follows:

$$D_t = Y_t - W_t n_{Yt} - I_{Kt} - T_{Et} e_t - \eta_1 \mu_t^{\eta_2} Y_t. \tag{8}$$

[12] The case of multiple sources of fluctuations is studied in Section 4 of the paper.

[13] This assumption reflects the notion that adjustment costs impact the model's short-term dynamics but have no long-run effect in a deterministic environment.

Relative to the model used by Kung and Schmid (2015), the potential for environmental regulation affects the problem of the representative firm by adding two additional terms. First, the fiscal authorities can decide to impose a tax on domestic emissions, which is denoted by T_{Et}. In the laissez-faire equilibrium, government intervention is absent, and the tax is set to zero. As we discuss in Section 2.10, under the optimal policy, the government is able to set the tax to a level that fully offsets the market failure present in the laissez-faire equilibrium.

Second, building on the class of models derived from integrated assessment models (IAMs) (e.g., Nordhaus, 2008), firms have access to an abatement technology which can be used to reduce the flow of emissions generated by the production process. We use a standard specification by which μ_t represents the fraction of emissions abated, where $1 \geq \mu_t \geq 0$. The parameter $\eta_1 \geq 0$ determines the maximum share of output that can be abated, while $\eta_2 > 0$ is the elasticity of the abatement cost with respect to the fraction of emissions abated. As we discuss in Section 2.5, firms in the laissez-faire equilibrium have no incentive to use this costly abatement technology, such that μ_t is equal to zero for all t.

Since the firm is owned by the representative agent, the flow of future expected dividends is discounted using m_t, the stochastic discount factor of the agent, which we derive in Section 2.3. As labor supply is completely inelastic and plays no role in this class of models (e.g., Jermann, 1998), we simplify the analysis by assuming that R&D is performed by managers.[14]

2.2 Climate Dynamics

Building on the literature on integrated assessment models (IAMs), our modeling approach utilizes the 'carbon cycle model' framework (e.g., Dietz, van der Ploeg, Rezai, and Venmans 2021), which traditionally encompasses multiple carbon reservoirs. However, in line with the recent work of Dietz and Venmans (2019),[15] we have adopted a simplified version of the carbon cycle featuring only one carbon reservoir. This streamlined specification allows for a more straightforward, closed-form expression of the optimal

[14]The implicit assumption is that households do not possess the skills to work in the R&D sector. Consequently, their compensation solely depends on the number of hours worked in the production sector. This assumption, which comes without loss of generality, simplifies the analysis by ensuring that firms do not need to be recapitalized if a large recession hits the economy. This issue can also be circumvented by adding monopolistic competition, which would come at the cost of adding another degree of freedom.

[15]Nordhaus (1991) or Golosov et al. (2014), among others, also employ a single carbon reservoir.

carbon tax, while still ensuring that the model dynamics remain consistent with climate dynamics.

The accumulation of carbon dioxide (CO_2) and other greenhouse gases (GHGs) in the atmosphere, resulting from human economic activity, is determined as follows:

$$x_{t+1} = (1 - \delta_X)\, x_t + \iota_X \left(e_t + e_t^* \right), \tag{9}$$

where x_{t+1} represents the stock of anthropogenic emissions in the atmosphere, δ_X is the quarterly rate of depreciation of the stock of carbon in the atmosphere, e_t is the flow of domestic emissions, expressed in gigatons of CO_2, and ι_X is a physical parameter converting CO_2 emissions into carbon.

As briefly mentioned in Section 2.1, we assume that the stock of carbon does not grow in the steady state. Given our focus on the United States (US), this assumption is justified by the evolution of emissions.[16] The stationarity of both emissions and the carbon stock ensures that the balanced growth path assumption is maintained, particularly with respect to climate damages that are not homogeneous in the production function.

The last term in Equation (9), e_t^*, represents the flow of emissions from the rest of the world, which is exogenously determined. These global emissions are also assumed to be stationary. Although it is soon to be able to draw definitive conclusions, evidence indicates that in China, the world's largest emitter, emissions are anticipated to peak in the near future (Chen, Xu, Gao, and Li, 2022). Additionally, substantial investments in clean energy production have contributed to a decoupling of GDP growth from CO_2 emissions in many countries, further suggesting that global emissions may eventually stabilize. Following Heutel (2012), we therefore assume that emissions from the rest of the world are non-cyclical and can be expressed as a proportion of U.S. emissions, $e^* = \varkappa e$, so as to capture the relative contribution of U.S. emissions to atmospheric carbon accumulation.

Domestic emissions result from production by firms:

$$e_t = (1 - \mu_t) \Upsilon_t Y_t \tag{10}$$

where μ_t represents the fraction of emissions that firms choose to abate, while Υ_t is the

[16]To motivate this modelling choice, in section B.2 of the online appendix, we formally test the significance of trends in macroeconomic time series. In contrast to other macroeconomic variables, CO_2 emissions in the US exhibit no time trend from 1973 to 2023, the period for which our empirical analysis is conducted.

rate at which emissions decouple from GDP. Unlike the DICE model (e.g., Nordhaus, 1991), our formulation endogenizes the decoupling rate, making it proportional to technological progress. Since output follows a time trend while emissions are stationary, balanced growth requires that the relationship between e_t and Y_t is driven by green technological progress, reflected in the decline of Υ_t. Specifically, green technological progress is tied to the economy's stock of human capital:

$$\Upsilon_t = \iota_E / H_t \tag{11}$$

where ι_E is parameter that the intensity of CO_2 emissions from the production of firms.

2.3 Households

Regarding preferences, we have employed a novel specification that combines two strands of literature. As is typically done in asset pricing, we follow the long-run risk approach pioneered by Bansal and Yaron (2004) and adopt Epstein-Zin-Weil (EZW) preferences (Epstein and Zin, 1989; Weil, 1989; Weil, 1990). Compared to Kung and Schmid (2015) and Kaltenbrunner and Lochstoer (2010), among others, our approach is unique in that it combines EZW preferences with habit formation.

Compared to the habit specification of Dew-becker (2014), our first distinction involves adopting an internal specification similar to that in Jermann (1998). The second difference is that we introduce slow-moving habits (e.g., Campbell and Cochrane, 1999) using the specification discussed in Fuhrer (2000). Finally, following the example of Stokey (1998), Acemoglu, Aghion, Bursztyn, and Hemous (2012), and others, the damage caused by emissions not only affects production but is also a source of disutility for households.

The utility function of the representative agent takes the following form:

$$U_t = \left[(1-\beta) \left(\frac{C_t}{x_t^{\nu}} - Z_t \right)^{\theta} + \beta \left[E_t \left(U_{t+1}^{1-\sigma} \right) \right]^{\frac{\theta}{1-\sigma}} \right]^{\frac{1}{\theta}}. \tag{12}$$

The evolution of the slow-moving habit stock, which we denote by Z_t, is given as follows:

$$Z_{t+1} = \varpi Z_t + (1-\varpi) \frac{C_t}{x_t^{\nu}}, \tag{13}$$

where $0 \leq \varpi \leq 1$. Our specification assumes that habits are formed over a composite good (e.g., Jaccard, 2014) consisting of both consumption and the stock of emissions.[17] Relative to Kung and Schmid (2015), our framework includes both habits as well as an environmental externality. The latter adds an additional parameter ν that quantifies the impact of emissions on utility.[18]

The time t budget constraint of the representative household is given as follows:

$$W_t n_{Yt} + s_t D_t + B_t \geq C_t + P_{Et}(s_{t+1} - s_t) + p_{Bt}(B_{t+1} - B_t) + T_{Ht}. \tag{14}$$

Revenues consist of labor income, denoted by $W_t n_{Yt}$, where W_t represents the wage rate. The capital income of our agents comprises two components. As owners of the representative firm, they receive dividend income, D_t, which depends on the number of shares, s_t, issued by the firm. Secondly, agents earn income from holding risk-free government bonds, receiving a fixed coupon payment normalized to 1 for each bond held B_t.

On the expenditure side, total income is primarily allocated to consumption, denoted by C_t. Each period, households also choose an amount of equity to purchase at price P_{Et}. Additionally, the quantity of one-period risk-free bonds purchased each period is denoted by B_{t+1}. The price of these one-period risk-free bonds, p_{Bt}, is inversely related to the risk-free rate r_{Ft}.[19] Finally, we assume that the government levies a lump-sum tax of T_{Ht}.

2.4 Government and Market Clearing

The government finances its spending through issuing a short-term risk-free bond and collecting taxes. Its budget constraint is

$$p_{Bt}(B_{t+1} - B_t) + T_{Ht} + T_{Et} e_t = G_t + B_t. \tag{15}$$

[17] Notice that with this specification, the model without habits is obtained by setting ϖ equal to 1. For $\varpi = 1$, we implicitly adjust the steady-state of Z, which becomes zero instead of one. This adjustment allows us to nest the CRRA utility function within the habit formation framework, providing a unified model that accommodates both preferences.

[18] This parameter, which modulates agents' aversion to emissions, is meant to capture plausible values for the social cost of carbon. Its calibrated value is discussed in detail in Section 3.1 while its moment implications are discussed in Section 3.7.

[19] In equation (14), the quantity of equities is assumed to be stationary, whereas the quantity issued of bonds has a time trend. This reflects the standard assumption made in the literature that equities are in fixed supply. In contrast, the quantity of bonds issued by the government is non-stationary.

Revenue comes from the sale of newly issued short-term government bonds $p_{Bt}B_{t+1}$ to households. Tax income is composed of two components: (i) the lump-sum tax levied on households T_{Ht}, and (ii) the revenues obtained from the implementation of an environmental tax on emissions $T_{Et}e_t$. Expenditures encompass public spending G_t and bond repayment B_t. Note that in the baseline case, $G_t = G$ is assumed to be fixed.[20]

The aggregate market clearing condition is obtained by consolidating the budget constraints of households (14), firms (7) and the government (15). It reads as follows:

$$Y_t = C_t + I_{Kt} + G_t + \eta_1 \mu_t^{\eta_2} Y_t. \tag{16}$$

2.5 The Competitive Equilibrium

We now describe a competitive equilibrium, where households and firms make independent economic decisions without government intervention. This is also known as a laissez-faire equilibrium. In this setting, social preferences for carbon emissions may differ across firms and households. We propose the following definition to characterize this equilibrium:

Definition 1 *A competitive equilibrium is a sequence of prices and quantities that satisfies three conditions:*

(i) Optimality: It solves the optimization problems of both households and firms. Households maximize lifetime utility as described in Equation (12), subject to constraints (13) and (14). Firms maximize profits as given by Equation (7), subject to constraints (1), (3), (4), (5), and (6).

(ii) Market Clearing: Prices clear markets, ensuring that the quantity supplied equals the quantity demanded in all markets, with the aggregate market clearing condition given by Equation (16).

(iii) Zero taxes on emission: In this equilibrium, government intervention is absent on climate distortion, implying a tax rate of zero on emissions, whereby $T_{Et} = 0$ for all t. Firms and households treat the existing stock of emissions, the evolution of which is determined by Equation (9), as a given variable.

[20]In section 4, we allow for time-varying public expenditures to capture cyclical changes in aggregate demand, while in subsection 3.8, the impact of distortionary fiscal policy is examined.

Compared to what would be socially optimal, the absence of carbon pricing in the laissez-faire equilibrium leads to excessive emissions. The competitive equilibrium is inefficient for two main reasons.

First, firms fail to internalize the fact that emissions increase the stock of carbon in the atmosphere, which in turn negatively affects their productivity. For a given atomistic firm, the effect of its own production on the aggregate stock of carbon is negligible. Consequently, firms have no incentive to use a costly abatement technology to reduce emissions, as the productivity benefits from emissions abatement are essentially zero at the individual firm level.

Second, firms do not account for the impact of their emissions on consumers. The fact that the stock of carbon is a source of disutility for households is not internalized in a competitive equilibrium where firms seek to maximize profits. A decentralized equilibrium in which the effect of emissions is not borne by the polluter, therefore, leads to an inefficient allocation of resources.

As a result, firms opt for a zero-abatement strategy in the laissez-faire equilibrium for all t:

$$\mu_t = 0. \tag{17}$$

2.6 The Centralized Equilibrium

As decentralized markets failed to deliver an efficient allocation of resources, we will now characterize the centralized equilibrium. The centralized equilibrium is the allocation that a social planner would implement under commitment. We will then show how the first-best allocation chosen by a social planner can be achieved by introducing a tax in the decentralized equilibrium described in Section 2.5.

We start by defining the centralized equilibrium. This equilibrium provides the benchmark against which the allocation obtained in the decentralized economy should be compared.

Definition 2 *The optimal policy problem for the social planner is to maximize total welfare in Equation* (12) *by choosing a sequence of allocations for quantities that satisfies Equations* (1), (3), (4), (5), (6), (9), (13), (14), *and* (16), *given initial conditions for the endogenous state variables* K_0, H_0, Z_0 *and* x_0.

In the centralized equilibrium, the law of motion for the stock of carbon in Equation (9) appears as an additional constraint in the planner's optimization problem. The negative externality on production and utility exerted by the accumulation of carbon in the atmosphere is therefore fully internalized. Since the externality resulting from carbon emissions is the only source of market failure, a first-best allocation of resources can be achieved in a centralized equilibrium.

2.7 The Stationary Economy

Given the human capital law of motion (3) and subject to the restrictions on preferences discussed by King et al. (1988), it is possible to derive a stationary equilibrium in which all variables grow at the same rate. In line with established methods in the field, we achieve this stationary equilibrium by normalizing trended variables by the rate of labor-augmenting technological progress, denoted as H_t. By dividing both sides of Equation (4) by H_t, the growth rate of technological progress, H_{t+1}/H_t, denoted as γ_t, is determined as follows:

$$\gamma_t = 1 + i_{Ht} \tag{18}$$

where i_{Ht} represents investment in human capital in the detrended economy.

In our setting, the effects of endogenous growth on the stationary equilibrium are fully encapsulated by γ_t. To underscore how the introduction of endogenous growth influences prices and quantities, we will now discuss the model's main implications with respect to the stationary economy.[21]

2.8 The Stochastic Discount Factor

With a combination of Epstein-Zin-Weil preferences and internal habits, the economy's stochastic discount factor (SDF) takes the following form:

$$m_{t+1} = \beta \gamma_t^{\theta-1} \left(\frac{u_{t+1}}{E_t\left(u_{t+1}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}} \right)^{1-\sigma-\theta} \left(\frac{x_{t+1}}{x_t} \right)^{-\nu} \left(\frac{(1-\beta)\left(\frac{c_{t+1}}{x_{t+1}^{\nu}} - z_{t+1}\right)^{\theta-1} - \varphi_{t+1}(1-\varpi)}{(1-\beta)\left(\frac{c_t}{x_t^{\nu}} - z_t\right)^{\theta-1} - \varphi_t(1-\varpi)} \right) \tag{19}$$

where m_{t+1}, u_{t+1}, c_t, and z_t denote the SDF, the value function in Equation (12), consumption, and the habit stock in the detrended economy. φ is the Lagrange multiplier associated with the habit stock in Equation (13).

What are the novel asset pricing implications? With balanced growth, any adjustment to the discount factor for growing economies (e.g., Kocherlakota, 1990) affects the

[21] The full derivation of the stationary equilibrium is provided and discussed in section A.3.2 of the online appendix.

continuation value of agents.[22] In our economy, the fact that γ_t is endogenously determined introduces a source of time variation in the SDF. Depending on the value of θ, the economy's growth rate γ_t can thus increase or reduce m_{t+1}. Moreover, as the stock of carbon x_t is a source of disutility, its evolution affects how agents discount future payoffs. Another key difference with respect to the climate risk literature is that the Lagrange multiplier φ_t, which reflects the impact of internal habits, also affects discounting and, hence, asset prices.[23]

2.9 Pricing Carbon

In the centralized equilibrium, agents are forced to internalize the impact of production on emissions. Unlike the competitive equilibrium, where the government takes the emissions trajectory as given, the planner in the centralized equilibrium determines the optimal sequence of emissions over time. It is important to note that this policy is implemented with commitment. The optimality condition with respect to x_{t+1}, expressed in gigatons of CO_2, yields the following formula for the optimal carbon price, υ_{Et}:

$$\upsilon_{Et} = E_t m_{t+1} \gamma_t \left(\iota_X \left(\nu \frac{c_{t+1}}{x_{t+1}} + \zeta \left(\varrho_{t+1} y_{t+1} + \phi_{t+1} i_{Ht+1} \right) \right) + (1 - \delta_X) \upsilon_{Et+1} \right) \tag{20}$$

where this price is expressed in trillions USD per gigaton of CO_2. One can convert this into the social cost of carbon (SCC) by expressing it in the form $SCC_t = 1{,}000\upsilon_{Et}$, which reflects a cost in dollars per ton of CO_2.

This condition closely mirrors the central asset pricing formula commonly found in finance textbooks. Here, the price of the carbon emissions stock is interpreted as the net present value of future marginal damages caused by emissions. While the stock of carbon is stationary, the carbon price exhibits a time trend. Consequently, in the stationary equilibrium, the SDF m_{t+1} must be adjusted and multiplied by γ_t.

The damage term consists of two main components. The first component reflects the cost borne by consumers, denoted by $\nu \frac{c_{t+1}}{x_{t+1}}$, which indicates the marginal disutility of carbon. Given a preference specification consistent with balanced growth, this term remains unaffected by the specific form of the utility function, as neither habit formation

[22] See Section 3.7 for a quantitative analysis of the case when endogenous growth becomes exogenous.

[23] Notice also that our SDF reduces to that typically used in the literature on long-run risk (e.g., Bansal and Yaron, 2004) if we remove both habit formation and the environmental externality by setting $\varpi = 1$ and $\nu = 0$.

nor the EZW parameters alter the disutility from carbon.

Since emissions affect both the consumption and production sides of the economy, the second component captures the marginal decline in productivity due to carbon accumulation, expressed as $\zeta\left(\varrho_{t+1}y_{t+1}+\phi_{t+1}i_{Ht+1}\right)$. Unlike in certain well-known specifications (e.g., Nordhaus, 2008; Golosov et al., 2014), this damage also extends to the R&D sector, influencing human capital accumulation. Furthermore, the damage is influenced not only by the quantities of output, y_t, and human capital investment, i_{Ht}, but also by their respective shadow prices, ϱ_{t+1} and ϕ_{t+1}.[24]

Finally, unlike a typical asset pricing formula, the continuation value, v_{Et+1}, accounts for the depreciation of the emissions stock, which occurs at a rate of δ_X per quarter.

2.10 Optimal Emissions Pricing

In the absence of carbon pricing, the allocation achieved in the competitive equilibrium is suboptimal. This market failure creates an opportunity for government intervention to address the environmental externality by ensuring that the decentralized allocation aligns with the social planner's optimal allocation.

Specifically, the government can introduce a tax on emissions to be paid by firms. In the stationary equilibrium, this tax is denoted τ_{Et}, where $\tau_{Et}=T_{Et}/H_t$. This policy tool can be viewed as a tax on carbon emissions, similar to a standard Pigouvian tax that aims to compel firms to internalize the social cost of carbon emissions on household utility and production. This approach corrects the market failure (i.e., the negative externality) by setting the tax equal to the price of carbon emissions.[25]

Comparing the social planner's solution to the competitive equilibrium, we propose the following:

Proposition 1 *The first-best allocation can be achieved by using the instrument τ_{Et} in the decentralized equilibrium. The first-best allocation of resources corresponding to the planner's problem can be attained by setting the tax on emissions such that*

$$\tau_{Et}=v_{Et}.$$

[24] ϱ and ϕ are the two Lagrange multipliers associated with constraints (1) and (4), respectively.

[25] An alternative interpretation is that the government creates a market for carbon emissions (i.e., a carbon-permits market). In this case, the government regulates the quantity of emissions. An equivalence between the tax and permit policies holds when the regulator has symmetric information about all state variables for any outcome under the tax policy and a cap-and-trade scheme (Heutel, 2012).

As discussed in the online appendix of the paper, setting the environmental tax τ_{Et} to v_{Et} ensures that the first-order conditions in both the competitive and centralized equilibria are identical.

Corollary 1 *In the decentralized equilibrium, setting the tax on emissions τ_{Et} to its optimal value ensures that firms choose a socially optimal level of abatement by setting μ_t such that:*

$$\mu_t = \left(\frac{v_{Et} \iota_E}{\eta_2 \eta_1} \right)^{\frac{1}{\eta_2 - 1}} \tag{21}$$

The optimal abatement level is determined by the price of carbon as well as the form of the abatement technology discussed in Section 2.2. Since abatement reduces profits, this condition also illustrates that firms will not be willing to bear this cost unless an enforcement mechanism is in place. Indeed, in the absence of carbon pricing, v_E equals zero, leading to zero abatement of emissions.

3 Moments Implications of Pricing Carbon

Conventional asset-pricing models are typically evaluated on their ability to jointly replicate realistic moments related to macroeconomic aggregates, such as consumption, output, and investment, as well as financial moments, such as the risk premium and risk-free rate. We estimate our baseline setup using the Simulated Method of Moments (SMM), following the double-step estimation method of Hansen and Singleton (1982).

Since this paper investigates the implications of replicating financial moments as a relevant factor for carbon pricing, we also estimate a CRRA specification, where most asset pricing moments are not matched. This CRRA framework is widely used in environmental economics, as in Heutel, 2012, Golosov et al., 2014, and Nordhaus, 2017. By comparing carbon policy outcomes in models that are and are not consistent with asset pricing data, we assess the implications of incorporating realistic asset pricing dynamics into carbon pricing models.

3.1 Calibration

The calibrated parameters are reported in Panel A of Table I. The calibration of the parameters related to business-cycle theory is standard: government spending as a percentage of GDP is set to 20 percent, capital intensity α to 0.33, and the share of hours worked $n_Y + n_H$ per day to 20 percent. To match the relative size of the R&D sector in output, we assume that the household spends 2.9% of their daily hours in the R&D sector. As in integrated assessment models, the levels of variables (such as output and emissions) and their corresponding steady states must have interpretable values in order to obtain a realistic accumulation of atmospheric carbon. The fixed level of productivity, denoted A, in the production function is meant to capture an output of $Y = 4.50$ trillion USD in 2015.

Table I: Model Parameters

This table reports model parameters. Panel A lists the calibrated parameter values. Panel B lists the parameters estimated using the simulated method of moments (SMM). We solve and simulate the model at a quarterly frequency based on 200 series of simulated data of 200 quarters. Column Baseline reports the estimation outcome under recursive-habits utility, while CRRA provides the outcome under standard constant relative risk aversion utility.

Panel A: Calibrated Parameters		
Parameter	Symbol	Value
Productivity level	A	4.1800
Spending-to-GDP ratio	G/Y	0.2000
Labor intensity	α	0.3300
Total labor supply	$n_Y + n_H$	0.2000
R&D labor demand	n_H	0.0290
World emissions factor	$\varkappa$	3.0000
CO_2 to carbon units	ι_X	0.2730
US CO_2 intensity (GtCO_2/trillion USD)	ι_E	0.4500
Carbon transfer rate to deep oceans	δ_X	0.0021
Initial carbon stock (GtC)	x_0	120.00
Abatement cost level	η_1	0.0200
Abatement cost curvature	η_2	2.6000
Damage elasticity	ς	0.00017
Carbon disutility	ν	0.16070

Panel B: Parameters Estimated via SMM			
Parameter	Symbol	Baseline (SD)	CRRA (SD)
Productivity standard deviation	$\text{std}(\varepsilon_{At})$	0.0154 (0.0008)	0.0118 (0.0001)
Productivity persistence	ρ_A	0.9717 (0.0068)	0.9985 (0.0075)
Steady state growth rate	g	0.59% (0.01%)	0.62% (0.02%)
Elasticity of capital adjustment costs	ϵ	4.0960 (0.0360)	0.2000 (0.1026)
Relative risk aversion	σ	5.9871 (0.6691)	1.1023 (0.0778)
Habits degree	ϖ	0.9408 (0.0018)	-
EZH utility curvature	θ	0.6872 (0.0315)	-
Discount factor	β	0.9844 (0.0025)	0.9877 (0.0011)
Capital intensity in R&D	ξ	0.1165 (0.0074)	0.1658 (0.0080)
Depreciation rate of capital	δ_K	0.0082 (0.0040)	-

The environmental component parameters of the model, when not estimated, are derived from the integrated assessment literature. The average emissions intensity, ι_E, is set to align with the U.S. post-WWII carbon intensity of $e/y = 0.45$ Gt CO_2. It is assumed that the rest of the world's emissions constitute approximately 75% of the total anthropogenic emission flow, leading to the emission factor $\varkappa = 3$. The conversion factor of CO_2 emissions into atmospheric carbon units is represented by the physical parameter $\iota_X = 3/11$. Regarding the continuation rate of carbon in the atmosphere, denoted δ_X, its value varies significantly across studies. We adopt a 120-year half-life for atmospheric carbon dioxide, with robustness checks for alternative values provided in the appendix.[26] The initial stock of anthropogenic CO_2 at the start of the sample in 1973 is set at $x_0 = 120$ $GtCO_2$.[27] For the damage function, the climate damage parameter γ is set to reflect a 15% GDP loss in the long run under the business-as-usual policy, resulting in $\nu = 1.25 \times 10^{-4}$. This value is based on Nordhaus (2017), though it is conservative compared to other estimates that suggest a GDP loss exceeding 60%. Finally, for the abatement cost function, we set $\eta_1 = 0.020$, which is close to the backstop price of carbon in the RICE-2011 model for the U.S., while the curvature parameter $\eta_2 = 2.6$ is sourced from the latest versions of the DICE model. To achieve realistic carbon price values, we adjust the carbon disutility parameter ν to 0.1607, resulting in a social cost of carbon of \$51 per ton in the business-as-usual scenario, consistent with the U.S. Biden administration's social cost of carbon.

3.2 Targeted Moments

The 10 targeted moments are reported in Table II.[28] Building on work by Jermann (1998), our framework is designed to match the following moments: the standard deviations of output, consumption, and investment, expressed in growth rates, as well as the

[26] This parameter value varies across studies. Heutel (2012) considers a lifetime of 82 years and provides a description of the range of carbon lifetimes, which lies between 19 to 139 years across studies. In particular, following Ita and Robert (1993) and Nordhaus (1991), we use a decay rate implying a half-life of 120 years. We use a value between Heutel (2012) and Nordhaus (1991) that translates to 480 periods in a quarterly frequency model ($\delta_X = 1/480$). As noted by Dietz and Venmans (2019), Integrated Assessment Models (IAMs) have certain limitations, including the handling of carbon's near-permanent lifecycle. A more accurate representation of climate dynamics would involve using a Transient Climate Response to Cumulative Emissions of Carbon Dioxide (TCRE) framework. However, since our primary objective is to isolate the impact of finance on the social cost of carbon compared to standard IAMs, we have deferred extending our model to the TCRE framework for future research. Nevertheless, we have conducted a robustness check assuming a carbon lifetime of up to 200 years, and the core results of our paper remain consistent.

[27] Because SMM/Bayesian estimations are sampling-based methods, our estimation includes an initial set of 50 burn-in periods spanning 1960-1973. Each chain of the SMM method initializes in 1960 with x_0 and randomly samples based on the distribution of innovations. The remaining samples, starting in 1973, are more likely to be representative of the true distribution of the model due to the burn-in phase. On average, over 200 chains, the stock of carbon in 1973 is 120 GtC. Further discussion, as well as the time path of carbon stock, is reported in Section D.2 of the online appendix.

[28] Appendix B provides in appendix a description of the data sources.

means and standard deviations of the risk-free rate and the equity premium. To inform the growth-related parameters, we include the mean growth rate of output and the mean ratio of capital in R&D to total capital, which helps estimate the capital intensity in the R&D sector. Since the model dynamics critically depend on the allocation of output between consumption and investment, we also include the average consumption-to-output ratio in the list of moments to match.

Compared to Jermann (1998), our estimation analysis includes additional moments related to long-run growth, as well as the consumption-to-GDP ratio and the volatility of macroeconomic variables. Because our model incorporates long-run risk and recursive preferences, it offers additional degrees of freedom to capture a broader set of moments more accurately.

3.3 Estimation of Model Parameters

The remaining set of parameters that are not calibrated are estimated. Given that our approach builds on the approach of Jermann (1998), we follow a similar strategy by using the simulated method of moments (SMM) to minimize the distance between a set of empirical moments and the corresponding simulated moments produced by the model. We implement the SMM routine according to standard practice in the field (e.g., Hansen and Singleton 1982; Kogan, Li, and Zhang 2023). Given a vector ψ of target moments in the data, we obtain parameter estimates using

$$\hat{p} = \arg\min_p \left(\psi - \frac{1}{S}\sum_{i=1}^{S}\hat{\psi}_i\left(p\right)\right)' W \left(\psi - \frac{1}{S}\sum_{i=1}^{S}\hat{\psi}_i\left(p\right)\right),$$

where $\hat{\psi}_i$ is the vector of moments computed in one out of S simulations of the model.[29]

Building on the work of Jermann (1998), we estimate the remaining 10 parameters that are not calibrated, as reported in Panel B of Table I. Similar to Jermann (1998), the first five parameters selected to maximize the model's ability to reproduce the target moments are the adjustment-cost parameter, the habit parameter, the subjective discount factor, the technology-shock standard deviation, and the shock-persistence parameter. We also introduce five new parameters relative to that study: the steady-state growth rate, the EZH utility curvature, the capital intensity in R&D, the depreciation rate of capital, and

[29]We simulate the model 200 times ($S = 200$), with each sample consisting of 200 observations, consistent with the size of the empirical sample. The optimization problem is solved using the simplex method to find the global minimum. We adopt the standard two-step variance-covariance estimator for W as proposed by Hansen and Singleton (1982).

the relative risk aversion.

Furthermore, to assess how the utility specification affects the model's ability to capture asset pricing moments, we estimate two versions of our model. The first, referred to as EZWH, features a recursive utility function with lifestyle habits. As discussed in the model section, this utility formulation sits at the intersection of Epstein and Zin (1989) and Jaccard (2014). Conversely, we also estimate a version of the model with standard constant relative risk aversion (CRRA), commonly used in most integrated assessment models, but known for its poor performance in replicating asset pricing moments with realistic levels of risk aversion.

Table II: SMM Targeted and Non-Targeted Moments under Alternative Preferences

This table presents the target moments from the data, along with their 5th and 95th percentiles. The two rightmost columns display the model-based results under recursive-habit preferences (EZWH) and power utility (CRRA). A star (*) next to a value indicates a targeted moment. Note that the set of targeted moments differs between the EZWH and power utility models.

Moment Comparison				
Moment	Symbol	Data *[0.025;0.975]*	EZWH	CRRA
Targeted Moments:				
Output growth mean	$E(100\,\Delta\ln Y)$	0.65 *[0.49;0.80]*	0.68*	0.71*
Output growth SD	$\mathrm{std}(100\,\Delta\ln Y)$	1.10 *[1.00;1.22]*	1.03*	1.06*
Consumption growth SD	$\mathrm{std}(100\,\Delta\ln C)$	1.01 *[0.92;1.12]*	1.03*	1.07*
Investment growth SD	$\mathrm{std}(100\,\Delta\ln I)$	2.12 *[1.93;2.35]*	2.11*	2.10*
Risk-free rate mean	$E(100\,r_F)$	0.13 *[0.01;0.24]*	0.09*	2.02
Risk-free rate SD	$\mathrm{std}(100\,r_F)$	0.81 *[0.74;0.90]*	0.79*	0.10*
Equity premium mean	$E[100\,[r_M - r_F])$	1.85 *[0.68;3.02]*	1.79*	0.00
Equity premium SD	$\mathrm{std}(100\,[r_M - r_F])$	8.40 *[7.65;9.32]*	8.41*	0.23
Consumption-to-output ratio	$E(100\,C/Y)$	61.8 *[61.6;61.9]*	61.7*	61.7*
R&D to total capital ratio	$E(100\,K_H/K_Y)$	17.0 *[16.8;17.2]*	17.0*	17.0*
Non-Targeted Moments:				
Cons. growth persistence	$\mathrm{AC}(\Delta\ln C)$	-0.12 *[-0.24;-0.01]*	-0.01	0.01
Tobins q growth	$\mathrm{std}(100\ \ln Q)$	7.93 *[7.32;8.64]*	8.69	0.42
Emission growth	$E(100\,\Delta\ln E)$	0.05 *[-0.90;1.00]*	0.00	0.00
Social cost of carbon \$/tCO_2	$E(1,000\,v_E)$	51.0	50.9	38.6

While the baseline model includes the 10 target moments mentioned earlier, a CRRA specification would typical fail to capture key features of financial data. To avoid the implausible parameter estimates that would arise from fitting CRRA to financial data, we preemptively exclude three finance-related moments from the CRRA estimation, as listed in Panel B, column CRRA of Table I. Following standard identification practices in SMM, we also remove three parameters from the estimation to maintain an equal number of instruments and target moments. Specifically, the habit parameter ϖ and the Epstein-Zin relative risk aversion parameter σ are excluded (as they do not apply to CRRA). Additionally, we exclude the depreciation rate of capital δ_K and instead calibrate it to 0.0082, the estimated value from the baseline model.

3.4 The Matching Performance of Each Utility Specification

Following standard practice in the asset pricing literature, we compare the model's implications—assuming the U.S. economy presently reflects the laissez-faire equilibrium—to the data. In Table II, the first and second columns show the estimated moments and their symbols, respectively. All results are presented at a quarterly frequency.

Consider first the baseline model, the Epstein-Zin-Weil model with slow-moving habit formation. This utility specification allows the model to reproduce all the targeted moments, with all simulated moments lying within the confidence intervals of the data. Compared to Jermann (1998),[30] who did not match the volatility of the risk-free rate, our model successfully captures both the low average risk-free rate and its standard deviation of 0.79 percentage points. Additionally, while Jermann (1998) produced a relatively high volatility for the market return, our model effectively captures the volatility of the returns on both risk-free and risky assets.

As shown in the lower part of the table, which reports moments that were not targeted, the main model specification captures salient features of the data in several key areas: consumption persistence, Tobin's q growth, emission growth, and the social cost of carbon as set by the Biden administration. All of these are consistent with observed data. In particular, the model's ability to capture fluctuations in Tobin's q demonstrates its robustness in incorporating investment dynamics, which is a significant improvement over traditional CAPMs. It has been widely documented that consumption exhibits a large first-order autocorrelation coefficient (e.g., Bansal and Yaron, 2004; Kung and Schmid, 2015). However, the COVID-19 outbreak, characterized by a significant decline in output, has disrupted this well-known strong first-order correlation pattern. Our model successfully captures this new feature of the US data.

In comparison, the power utility model with long-run risk (referred to as CRRA in Table II) performs relatively well in capturing the key features of macroeconomic variables. However, as expected, it fails to replicate asset pricing moments, such as the mean and volatility of the equity premium and risk-free rates, as well as non-targeted

[30]Note also that Bansal and Yaron (2004) demonstrate insufficiently large fluctuations in the risk-free rate with recursive utility and long-run risk.

moments like Tobin's q. This outcome is unsurprising, given that the model is not calibrated to financial data, and capturing these moments would require implausibly large risk aversion coefficients (Mehra and Prescott, 1985).

3.5 Impacts of Finance on Optimal Carbon Pricing

In this subsection, we examine the effects of *finance*—specifically, the role of asset pricing—on the optimal pricing of carbon emissions in a version of the model that captures key features of financial data. The columns labeled *EZWH Carbon Price* and *CRRA Carbon Price* in Table III present the simulated moments when the optimal carbon price is implemented.

A significant finding is that a realistic discount rate, grounded in financial moments, substantially impacts the price of carbon $E(v_E)$, as shown in the *EZWH* columns in Table III. The price is 32% higher than that obtained under the power utility function. This has crucial implications for mitigation policies, especially in frameworks where risk plays no role in the SDF (e.g., Heutel, 2012; Golosov et al., 2014; Nordhaus, 2017). An environmental model consistent with realistic discounting necessitates a 25% increase in mitigation efforts $E(\mu)$. This result aligns with the climate risk literature, which suggests that uncertainty is a significant factor in determining the stance of environmental policy.

The underlying reason for this effect lies in the relative change in the discount rate provided by EZWH preferences. Two mechanisms, driven by the financial component of the model, contribute to the increase in the carbon tax. Both mechanisms relate to the determination of the SDF. First, the early resolution of uncertainty, due to recursive preferences, alters the pricing of future risks in present terms. Investors' aversion to uncertainty increases their demand for insurance against potential declines in future consumption, which raises the SDF and, consequently, the SCC. Second, the impact of consumption habits on the SDF, as discussed by Abel (1990) and Campbell and Cochrane (1999), demonstrates that agents become more risk-averse during periods when consumption is low relative to past levels. In response, investors increase their precautionary savings to buffer against potential future declines in consumption, further raising the SDF.

Given the long lifetime of carbon, the social planner prices atmospheric carbon similarly to a financial asset with a claim on marginal climate damages. By applying the same discounting principles as those used for financial assets, the pricing of carbon is significantly influenced by finance. The increase in the SDF, driven by the model's ability to generate a realistically low interest rate, amplifies the relative weight of future marginal climate damages, necessitating a more stringent environmental policy. Thus, the effects of *finance*—specifically, the role of asset pricing—on the optimal pricing of carbon emissions suggest a higher social cost of carbon, much larger than those derived from power utility frameworks like that of Nordhaus (2017). This finding corroborates the conclusions of Cai and Lontzek (2019) and Van Den Bremer and Van Der Ploeg (2021), though our framework is distinguished by its consistency with all asset pricing and its inclusion of slow-moving habits.

Table III: Moments under laissez-faire versus carbon price policy

This table compares the moments in the laissez-faire equilibrium and the economy under the optimal carbon policy specifications. The welfare cost of fluctuations is calculated as the fraction of output that would need to be added or subtracted to make the stochastic economy equivalent to the deterministic one.

Moment	Symbol	EZWH		CRRA	
		Laissez-faire	Carbon Price	Laissez-faire	Carbon Price
Targeted Moments:					
Output growth mean	$E(100 \ln \Delta Y)$	0.68*	0.72	0.71*	0.73
Output growth SD	$\mathrm{std}(100 \ln \Delta Y)$	1.05*	1.09	1.09*	1.09
Consumption growth SD	$\mathrm{std}(100 \ln \Delta C)$	1.03*	1.01	1.07*	1.08
Investment growth SD	$\mathrm{std}(100 \ln \Delta I)$	2.11*	1.88	2.10*	2.07
Consumption-to-output ratio	$E(100\, C/Y)$	61.7*	60.3	61.7*	59.0
R&D to total capital ratio	$E(100\, K_H/K_Y)$	17.0*	17.5	17.0*	17.3*
Risk-free rate mean	$E(100\, r_F)$	0.09*	0.35	2.02	2.05
Risk-free rate SD	$\mathrm{std}(100\, r_F)$	0.79*	0.71	0.10*	0.10
Equity premium mean	$E(100\, [r_M - r_F])$	1.79*	1.53	0.00	0.00
Equity premium SD	$\mathrm{std}(100\, [r_M - r_F])$	8.41*	7.31	0.23	0.42
Non-Targeted Moments:					
Abatement share	$E(\mu)$	0.00	0.60	0.00	0.75
Carbon stock mean (GtC)	$E(x)$	310	165	306	186
Carbon price in $/tCO$_2$ mean	$E(1000\, v_E)$	0.00	72.3	0.00	51.0
Carbon price in $/tCO$_2$ SD	$\mathrm{std}(1000\, v_E)$	0.00	15.7	0.00	4.51
Welfare mean	$E(U)$	0.15	0.18	-746	-738
Welfare cost (% GDP)	$E(\mathbb{U}_{\%})$	0.60	0.55	0.09	0.08
Dividend growth SD	$\mathrm{std}(100 \ln \Delta D)$	0.69	1.37	0.70	0.92
Bond premium mean	$E(100\, [r_B - r_F])$	1.99	1.81	0.73	0.75
Carbon price-cons. beta	$\mathrm{cov}(\hat{v}_E, \hat{c})/V(\hat{c})$	0	4.95	0	0.75

3.6 Procyclical Carbon Price and Asset Returns

A key difference in this framework compared to the existing literature is that the link between the optimal tax and the SDF breaks the classic dichotomy between macroeconomics and finance (e.g., Cochrane, 2017). This macro-finance union is evident in the implementation of a carbon price, which influences asset valuation. The equity premium declines from 1.79 to 1.53 percent on quarterly basis, while the mean risk-free rate more than triples, increasing from 0.09 to 0.35 percent. Annually, this yields about a 1 percent change in each rate.

Why does environmental policy affect asset valuation? This effect is directly related to the procyclical nature of the carbon tax.[31] It is well established in the literature with power utility (e.g., Heutel, 2012; Golosov et al., 2014) that the carbon tax is inherently procyclical. Since the carbon tax, representing a sum of marginal climate damages, is proportional to output, it naturally exhibits procyclicality. Under power utility, a 1 percent increase in consumption leads to a 0.75 percent rise in the carbon tax to curb the corresponding boom in carbon emissions, as measured by $cov(\hat{v}_E, \hat{c})/V(\hat{c})$.

In an economy with realistic discounting, the SDF exhibits a significant time variation and heightened procyclicality that increases by a factor of 5 compared to power utility. This means that the carbon price will rise significantly more during booms and fall more sharply during recessions, thereby reducing aggregate risk.

This increased procyclicality affects investors and breaks the macro-finance dichotomy. To illustrate how reducing uncertainty about consumption fluctuations affects investors, Table III reports our measures of welfare and the cost of business cycle fluctuations, denoted by $E(\mathbb{U})$ and $E(\mathbb{U}_{\%})$, respectively.[32] By smoothing consumption fluctuations, a carbon tax with realistic discounting reduces aggregate risk, as evidenced by a 10 percent reduction in the welfare cost of fluctuations. Investors exhibit a reduced precautionary saving, leading to a 1% annual increase in the risk-free rate.

The lowered risk premia can be attributed to changes in either the SDF or dividends. To isolate the effect of the SDF on the valuation of risky assets, consider the risk premium on bonds, which differs from the market rate due to the absence of dividends in their payoff. The risk premium on bonds decreases by 0.18 percent quarterly, approximately the same as the reduction in the risk premium on dividend claims. This effect confirms that the smoothing effect of the carbon tax also contributes to a reduction in risk premia.[33]

[31] To measure this procyclicality, we calculate the beta between the log of the carbon tax and the log of consumption using a linear regression $\hat{v}_{Et} = \alpha + \beta \hat{c}_t + \epsilon_t$, where β measures the degree of co-movement between the carbon tax and changes in consumption, expressed as log deviations from the steady state.

[32] Since our model separates risk aversion from the intertemporal elasticity of substitution, interpreting welfare costs computed directly from the utility function can be challenging due to strong local nonlinearities (e.g., Tallarini, 2000). Following Jaccard (2024), we compute welfare costs from the production side. This approach determines the fraction of output that would be gained or lost when comparing the stochastic and deterministic economies, providing reasonable values of the welfare cost. As demonstrated by Tallarini (2000), our measured welfare cost of business cycle fluctuations is significantly higher (0.6% of GDP versus 0.09% in CRRA) in models capable of generating risk premiums of a realistic magnitude. Because investors are risk-sensitive, their precautionary motive is reinforced by recursive utility and habit formation.

[33] The dominant role of the SDF in determining risk premia is further confirmed by the increased

In our general equilibrium model, the smoothing effect of the carbon tax reduces the compensation required by investors to hold risky assets and decreases the need for precautionary saving.

3.7 Intuitions: the Core Mechanisms of Carbon and Asset Pricing

This paper introduces two key findings that contrast with the existing literature: the pronounced procyclicality of carbon pricing and the reduction in risk premia following the implementation of a carbon tax. To investigate which channels drive these effects, Table IV considers the EZWH preferences estimated under the simulated method of moments (SMM). We systematically deactivate different mechanisms and measure their respective impacts on core moments relevant to our research questions.

Table IV: Sensitivity of moments under laissez-faire (LFP) versus carbon price (CBP) policy

This table compares the moments in the laissez-faire (LFP) equilibrium and the economy under the optimal carbon policy (CBP) specifications. All moments are reported per quarter. $\hat{c}$ denotes the log deviation of consumption from the steady state, r_F the risk-free rate, ep the equity premium $r_M - r_F$, v_E the effective carbon tax expressed in \$ per tCO_2 by multiplying by factor 1,000, and $cov(\hat{v}_E, \hat{c})/V(\hat{c})$ is measured by coefficient β estimated in the OLS regression $\hat{v}_E = \alpha + \beta\hat{c} + \epsilon$. Each moment is computed on 200 parallel chains with 200 observations per chain. All variants of the model have the same calibration, differing only in a single restriction as reported in the first column of the table.

	std(100 $\hat{c}$) LFP [*CBP*]	E(100 r_F) LFP [*CBP*]	E(100 ep) LFP [*CBP*]	E(1000 v_E) LFP [*CBP*]	std(1000 v_E) LFP [*CBP*]	$\frac{\text{cov}(\hat{v}_E,\hat{c})}{\text{V}(\hat{c})}$ LFP [*CBP*]
Baseline						
EZWH	4.75 [*4.48*]	0.09 [*0.35*]	1.79 [*1.53*]	0.00 [*72.3*]	0.00 [*15.7*]	0.00 [*4.95*]
Utility functions						
EZW ($\varpi = 1$)	7.75 [*7.65*]	1.73 [*1.74*]	0.13 [*0.14*]	0.00 [*60.1*]	0.00 [*5.19*]	0.00 [*0.88*]
Habits ($\theta = 1 - \sigma$)	5.18 [*4.90*]	1.21 [*1.44*]	1.75 [*1.68*]	0.00 [*34.5*]	0.00 [*11.6*]	0.00 [*7.49*]
CRRA ($\theta = 1 - \sigma$, $\varpi = 1$)	6.57 [*6.48*]	4.39 [*4.49*]	0.70 [*0.68*]	0.00 [*14.2*]	0.00 [*2.29*]	0.00 [*3.03*]
Tax schemes						
Fixed tax ($v_E = \bar{v}_E$)	4.75 [*4.72*]	0.09 [*0.24*]	1.79 [*1.68*]	0.00 [*37.4*]	0.00 [*0.00*]	0.00 [*0.00*]
Income/dividend tax (adjusted T_{H_t})	4.75 [*4.48*]	0.09 [*0.35*]	1.79 [*1.53*]	0.00 [*72.3*]	0.00 [*15.7*]	0.00 [*4.95*]
Income/dividend tax (adjusted G_t)	5.15 [*4.89*]	-0.23 [*0.04*]	2.09 [*1.81*]	0.00 [*64.98*]	0.00 [*15.29*]	0.00 [*4.91*]
Macroeconomic assumptions						
Exogenous growth ($\gamma_t = \bar{\gamma}$)	6.06 [*5.78*]	-1.1 [*-0.9*]	3.37 [*3.07*]	0.00 [*46.2*]	0.00 [*15.7*]	0.00 [*5.94*]
Neutral env. pref. ($\nu = 0$)	4.83 [*4.72*]	0.19 [*0.30*]	1.74 [*1.62*]	0.00 [*27.8*]	0.00 [*6.63*]	0.00 [*4.75*]
Household heterogeneity	4.83 [*4.49*]	-0.0 [*0.30*]	1.92 [*1.60*]	0.00 [*79.4*]	0.00 [*17.5*]	0.00 [*5.03*]

It is noteworthy that, in any formulation, the implementation of a carbon tax leads to a reduction in consumption volatility. This is indicated by the first column of $std(\hat{c})$ when comparing the laissez-faire policy (LFP) against the carbon price policy (CBP). However,

volatility of dividend growth resulting from carbon price policy. The heightened perceived risk and uncertainty associated with investments would typically require a larger premium as compensation. Since risk premia are instead reduced, one can conclude that the effect from the SDF dominates.

the procyclical nature of financial dynamics is primarily driven by slow-moving habits rather than recursive preferences. This is evident in the last column $cov(\hat{v}_E, \hat{c})/V(\hat{c})$, where the co-movement is consistently greater under habit formation (EZWH and habits alone) compared to other formulations of the utility function. The time variation in the SDF induced by the habit stock amplify the joint fluctuations in consumption and the carbon tax.

The macro-finance union observed in our results originates from the presence of consumption habits, as shown in Table IV, since this effect only manifests under such preferences. The impact of consumption habits on the SDF, as discussed by Abel (1990) and Campbell and Cochrane (1999), demonstrates that agents become more risk-averse during periods when consumption is low relative to past levels. A strongly procyclical carbon price helps mitigate fluctuations in marginal utility, reducing investors' risk aversion. This decrease in consumption volatility lessens the precautionary savings motive among investors, leading to a higher risk-free rate. Furthermore, by stabilizing consumption patterns, overall economic risk is diminished, resulting in lower risk premia.

Additionally, long-run risk (LRR) influences the SDF by moderating the co-movement between consumption and the carbon tax, as it must also align with the long-run technological trend.[34] In the presence of a fixed technological trend (as shown in the row labeled "exogenous growth" in Table IV), procyclicality is further reinforced, with $cov(\hat{v}_E, \hat{c})/V(\hat{c})$ approaching 6. The environmental tax smooths fluctuations in the marginal utility of consumption, thereby amplifying the effect of the carbon price on asset prices.

Finally, environmental preferences, denoted by ν, play a significant role in shaping asset prices in the laissez-faire economy. By comparing the baseline case with the neutral environmental preference rows in Table IV, one can observe how climate damage alone as unique distortion affect the moments. The presence of climate disutility, with $v > 0$, significantly lowers the risk-free rate due to the precautionary motive, while the risk premium increases.

3.8 Consumption smoothing and fiscal policies

The mechanism through which climate policy affects asset prices primarily hinges on the smoothing effect of tax policy. To isolate the contribution of an exacerbated

[34]LRR also impacts the risk premium by reducing it. Assets negatively correlated with long-run consumption growth (i.e., assets that provide returns when long-term consumption expectations worsen) serve as hedges and therefore tend to have lower risk premia. In the absence of stochastic long-term growth, this hedging benefit disappears thus explaining the rise up to 3% of the risk premium.

procyclical carbon tax on asset returns, we fix the tax at its steady-state level, $v_E = \bar{v}_E$, as shown in Table IV.[35] The results indicate that under an acyclical carbon price, the reduction in consumption volatility is minimal, leading to only a modest reduction in risk premia and a slight increase in the risk-free rate. Therefore, much of the asset pricing effect is confirmed to emanate from the high procyclicality itself, rather than simply a permanent change in the level of the carbon tax.

To further investigate how fiscal policy assumptions affect our main results, we will expand the fiscal block to include income and dividend taxes.[36] We will examine how income and dividend taxes interact with carbon pricing and asset valuations.[37] To explore this, we simulate a scenario where both income and dividend taxes are set at 15%, corresponding to the average rates in the U.S. for 2021 (based on U.S. Federal Income Tax data). The updated household and government budget constraints are as follows:

$$(1-\tau_w)W_t n_{Y_t} + (1-\tau_d)s_t D_t + B_t \geq C_t + P_{E_t}(s_{t+1} - s_t) + p_{B_t}(B_{t+1} - B_t) + T_{H_t} \quad (22)$$

$$p_{B_t}(B_{t+1} - B_t) + T_{H_t} + T_{E_t} e_t + \tau_w W_t n_{Y_t} + \tau_d s_t D_t = G_t + B_t \quad (23)$$

where τ_w and τ_d represent the income and dividend taxes set at 0.15 in the laissez-faire economy. Since government debt is in net zero supply, any change in the fiscal block affects other components of the government budget constraint. The introduction of a carbon tax provides additional revenue to the fiscal authority but also reduces the tax base for other revenues. Therefore, we allow for a more realistic fiscal budget by adjusting either household lump-sum transfers, T_H (see Income/Dividend Tax row with T_H in Table IV), or public spending, G (see Income/Dividend Tax row with G).[38]

These changes in fiscal spending may impact consumption smoothing and alter the reduction in risk premia observed in the baseline model. Similar to the fixed carbon tax scenario, a 15% income and dividend tax does not offset the consumption smoothing

[35] Note that the fixed carbon tax is detrended, implying that the carbon tax grows at a constant rate proportional to output growth in the growing economy.

[36] Barrage (2020) explores the impact of carbon taxes on fiscal revenues, noting that carbon taxation can potentially shrink the tax base for other revenue streams.

[37] A capital tax could also be introduced; however, since firms act as their own suppliers, it would not directly impact the household's budget constraint. Similarly, a consumption tax could be implemented, but its effects would cancel out in the Euler equation, rendering it ineffective. Lastly, while a carbon tax could be applied to labor costs, given the inelasticity of labor supply, it would not affect equilibrium wages.

[38] When lump-sum transfers are allowed to adjust, they increase by approximately 3% in the steady state under the optimal carbon policy compared to the laissez-faire scenario. In contrast, public spending increases by around 5%.

effects introduced by a carbon price. First, due to the permanent income hypothesis in our framework, changes in lump-sum transfers do not affect the Euler equation or the resulting moments. In contrast, in the scenario where government spending adjusts to carbon tax revenues, the volatility of consumption increases, thereby amplifying the precautionary savings motive and raising the risk of holding risky assets. Overall, the core findings of the paper remain robust even when accounting for a more comprehensive fiscal block.

3.9 Consumption smoothing and households heterogeneity

We extend our analysis to incorporate heterogeneity, acknowledging that representative-agent models does not capture the wealth distribution. To address this, we consider an economy with heterogeneous agents, consisting of a representative saver (sa) and a representative spender (sp). The representative saver earns wages, receives dividends, and earns returns on bonds, while the representative spender collects wages for labor provided and consume it. Both agents pay lump-sum taxes or receive lump-sum transfers from the government. Their respective budget constraints are given by:

$$W_t n_{Y_t}^{sa} + s_t D_t + B_t \geq C_t^{sa} + P_{E_t}(s_{t+1} - s_t) + p_{B_t}(B_{t+1} - B_t) + T_{H_t}^{sa} \tag{24}$$

$$W_t n_{Y_t}^{sp} \geq C_t^{sp} + T_{H_t}^{sp} \tag{25}$$

In this framework, the pricing kernel for carbon no longer coincides with the pricing kernel for assets (from the saver). The planner seeks to maximize the discounted lifetime aggregate utility of consumption, $u(C_t)$, where $C_t = \chi_c C_t^{sa} + (1 - \chi_c) C_t^{sp}$. Here, χ_c represents the share of savers in the economy, set at 70%, in line with Kaplan, Moll, and Violante (2018) for the U.S. economy. Transfers are distributed proportionally according to the population share of each representative agent, with $T_{Ht} = \chi_c T_{Ht}^{sa} + (1 - \chi_c) T_{Ht}^{sp}$.

The last line of Table IV presents the results when we introduce heterogeneity, reflecting an economy where asset pricing is concentrated among a limited number of investors. Because a significant fraction of agents has limited access to financial markets to smooth consumption, they primarily consume their income, increasing aggregate risk. As a result, the risk of holding risky assets grows for savers, while their demand for safe assets rises. This leads to a 10% increase in both the level and volatility of the carbon price compared to the representative-agent case, thereby reinforcing our core findings. In this heterogeneous economy, the carbon tax becomes more procyclical, amplifying the asset pricing effects of climate policy.

4 The Cyclical Implications of Carbon Pricing

In the previous section, we discussed how finance implies significant procyclicality of the carbon price when the utility function exhibits habit formation. Is this class of utility function favored by the data? To answer this question, we estimate the model not only on specific asymptotic targeted moments (as in SMM), but also on the conditional realization of shocks through full-information methods (e.g., Smets and Wouters, 2007). The advantage of this methodology is that it aims to replicate the entire realization of the data, allowing us to discern which utility function best replicates the data from a statistical standpoint. The following subsections discuss the methodology employed for the estimation of the nonlinear model, the description of prior and posterior distributions, and a quantitative analysis based on the sequences of shocks and set of parameters estimated through the Bayesian methodology.

4.1 Methodology

In order to accurately measure higher-order effects related to risk-sensitive preferences, we consider a third-order approximation of the decision rules of our model, based on the laissez-faire version of the economy. Estimating dynamic general equilibrium models using higher-order approximations remains a challenge as the nonlinear filters required to form the likelihood function are computationally expensive. The paper relies on the inversion filter, as the latter offers a computationally affordable alternative to apply nonlinear models to data.[39] To allow the recursion, this filter requires that the number of fundamental shocks be equal to the number of observable variables.[40] Because our sample includes five observable variables, we introduce four additional sources of fluctuations in the model in addition to the TFP shock. Those shocks take the form $\log(\varepsilon_{Jt}) = \rho_J \log(\varepsilon_{Jt-1}) + \eta_{Jt}$ for $j = \{B, G, E, D\}$.

A first shock is introduced to the household discount factor to capture exogenous demand shifts (see Basu and Bundick, 2017). Utility $\varepsilon_{Bt} C_t / x_t^{\nu}$ both in welfare function and habit stock is multiplied by ε_{Bt} in Equation 12 and 13. A second shock affects public spending, with $G_t = G\varepsilon_{Gt}$ in Equation 15 and 16, where G is the structural component

[39] Initially pioneered by Fair and Taylor (1987), this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g., Kalman or particle), the inversion filter relies on an analytic characterization of the likelihood function. Kollmann (2017) provided the first application of the inversion filter to second- and third-order approximations to the decision rules in a rational-expectations model.

[40] Note that for linearized models, this restriction is standard such as in Smets and Wouters (2007).

of public spending and ε_{Gt} its cyclical counterpart (see Smets and Wouters, 2007). A third shock is introduced to domestic emissions in Equation 10, which is now calculated as $(1-\mu_t)\Upsilon_t Y_t \varepsilon_{Et}$ in order to capture cyclical changes in the carbon intensity of the US economy. Finally, the last shock is a dividend shock (see Bansal and Yaron, 2004) that captures additional variability in the distribution of dividends. It is assumed that this shock follows a unit root $\rho_D = 1$ so as to capture drifts in the diffusion of dividends.

Our data, when shared, are consistent with those used in the SMM section, covering the same sample period. Data sources and transformations are detailed in Appendix B. The measurement equations linking our model to the data are

$$\begin{bmatrix} \text{Real output growth} \\ \text{Consumption-to-GDP ratio} \\ \text{CO}_2 \text{ emissions growth} \\ \text{Real risk-free interest rate} \\ \text{Ex post equity premium} \end{bmatrix} = \begin{bmatrix} \Delta \log (Y_t) \\ C_t / Y_t \\ \Delta \log (e_t) \\ r_{Ft} \\ r_{Mt} - r_{Ft-1} \end{bmatrix}. \tag{26}$$

4.2 Parameter Estimation

As for the SMM estimation, we use the same calibration as in Table I and the same set of parameters to be estimated. Following the standard practice in estimated macroeconomic models, we employ Bayesian methods by augmenting the likelihood function with priors so as to address misspecification issues. Table V summarizes the prior and posterior distributions of the structural parameters for the U.S. economy.

Specifically, the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2. For the standard deviation of shocks, we use an inverse gamma distribution with a mean of 0.001 and a standard deviation of 0.0033, similarly to Christiano, Motto, and Rostagno (2014). The steady-state percent growth rate of productivity g follows a gamma distribution with a mean of 0.01 and a standard deviation of 0.001 in order to match the average growth rate of the US economy. The habit parameter ϖ is assigned a beta distribution with a mean of 0.5 and a standard deviation of 0.08. The elasticity of Tobin's Q to the investment-capital ratio ϵ follows a gamma distribution with a mean of 5 and a standard deviation of 0.5, consistent with Jermann (1998). The rest of the macroeconomic parameters have priors that are consistent with parameter estimates in SMM. We assume that the capital intensity in the R&D sector ξ follows a beta distribution with a mean of 0.1 and a standard deviation of 0.04, consistent with its value estimated in SMM. Additionally, the discount factor β follows a beta

Table V: Prior and posterior distributions of structural parameters of the model with recursive preferences and habit formation

This table reports the means and the 5th and 95th percentiles of the posterior distributions drawn from height parallel Markov Chains Monte Carlo of 10,000 iterations each. The sampler is the Metropolis-Hasting algorithm with a jump scale factor, so to match an average acceptance rate close to 25–30 percent for each chain. In the column Shape, $\mathcal{B}$ denotes the beta, $\mathcal{IG}_2$ the inverse gamma (type 2), $\mathcal{IG}$ the inverse gamma (type 1), $\mathcal{G}$ the gamma, $\mathcal{N}$ the Gaussian.

		Prior Distribution			Posterior Distribution		
		Shape	Mean	Std	Mode	Mean [5%:95%]	
Panel A: Shock processes							
Productivity shock	$std(\eta_{At})$	$\mathcal{IG}_2$	0.001	0.0033	0.0128	0.0130	*[0.0118:0.0144]*
Preference shock	$std(\eta_{Bt})$	$\mathcal{IG}_2$	0.001	0.0033	0.0091	0.0094	*[0.0085:0.0104]*
Dividend shock	$std(\eta_{Dt})$	$\mathcal{IG}_2$	0.001	0.0033	0.0935	0.0938	*[0.0870:0.1011]*
Emission shock	$std(\eta_{Et})$	$\mathcal{IG}_2$	0.001	0.0033	0.0219	0.0222	*[0.0205:0.0242]*
Spending shock	$std(\eta_{Gt})$	$\mathcal{IG}_2$	0.001	0.0033	0.0229	0.0239	*[0.0194:0.0284]*
Persistence productivity	ρ_A	$\mathcal{B}$	0.5	0.2	0.9751	0.9750	*[0.9668:0.9842]*
Preference presistence	ρ_C	$\mathcal{B}$	0.5	0.2	0.9496	0.9517	*[0.9402:0.9651]*
Emissions presistence	ρ_E	$\mathcal{B}$	0.5	0.2	0.9648	0.9625	*[0.9266:0.9870]*
Sprending presistence	ρ_G	$\mathcal{B}$	0.5	0.2	0.7422	0.7487	*[0.7174:0.7817]*
Panel B: Structural parameters							
Steady state growth	g	$\mathcal{G}$	0.01	0.001	0.0057	0.0057	*[0.0052:0.0065]*
Capital in R&D	ξ	$\mathcal{B}$	0.1	0.04	0.0733	0.0738	*[0.0504:0.0997]*
Discount factor	β	$\mathcal{B}$	0.985	0.001	0.983	0.9834	*[0.9818:0.9850]*
Investment cost	ϵ	$\mathcal{G}$	5	0.5	2.4009	2.4053	*[2.3822:2.4364]*
Capital depreciation	δ_K	$\mathcal{B}$	0.01	0.005	0.0106	0.0102	*[0.0077:0.0128]*
Utility curvature	θ	$\mathcal{N}$	0	0.1	0.2313	0.2012	*[0.1044:0.2970]*
Risk aversion	σ	$\mathcal{IG}$	4	3	2.8758	2.8844	*[2.8571:2.9126]*
Habit smoothing	ϖ	$\mathcal{B}$	0.5	0.08	0.9507	0.952	*[0.9387:0.9620]*

distribution with a mean of 0.985 and a standard deviation of 0.001, while the capital depreciation rate δ_K also follows a beta distribution with a mean of 0.01 and a standard deviation of 0.005, allowing their respective posterior values to remain within the intervals found under SMM.

Regarding the priors for intertemporal elasticity of substitution (IES) and risk aversion, we follow the survey by Cai and Lontzek (2019) on the usual calibration practices in the field. That survey indicates that IES usually lies between 1 and 2, so we impose a gamma distribution on the utility curvature parameter θ. We assume a mean of 0 and a standard deviation of 0.1 so as to keep the posterior values within the usual range considered in this literature. In contrast, risk aversion σ typically lies between 2 and 10 in finance-related literature but can be much higher in macroeconomic literature (e.g., Rudebusch and Swanson, 2012 and Basu and Bundick, 2017). To cover a parameter range consistent with both lines of research, we impose an inverse gamma distribution with a mean of 4 and a standard deviation of 3. The inverse gamma distribution is sufficiently diffuse in the right tail, allowing the data to accommodate both relatively low and high values (see Barro, 2009 and Basu and Bundick, 2017, respectively).

The priors and their corresponding posterior distributions for the structural parameters are detailed in Table V. In addition to prior distributions, Table V reports the means and the 5th and 95th percentiles of the posterior distributions for the model with recursive preferences and habits.[41] Our estimates of the structural parameters previously examined by Smets and Wouters (2007) align mostly with their findings. This includes the persistence of productivity and spending shocks, as well as the growth rate of 0.57%. For the elasticity of Tobin's Q to the investment-capital ratio ϵ, we find a posterior mean of 2.4, which is lower than that found by Jermann (1998) but falls within the accepted interval of values for his SMM matching. In addition, the discount factor, capital depreciation, habit smoothing, and capital intensity in R&D are remarkably similar to the results of the SMM, as their estimated values in SMM lie within the 95% confidence interval of the Bayesian estimation. In contrast, there are two notable differences: utility curvature and risk aversion. The Bayesian estimation implies a relatively lower IES, close to 1.3, and a relatively lower risk aversion that is consistent with the acceptable parameter range in Mehra and Prescott (1985). Because the risk aversion is above IES, consumers prefer to resolve uncertainty about future outcomes sooner by commanding a premium for holding assets that are subject to long-term risks. Note also that our economy exhibits an IES larger than 1 and long-run risk; agents demand large equity risk premia because they fear that a reduction in economic growth prospects will lower asset prices, as documented by Bansal and Yaron (2004).

4.3 The Empirical Dominance of Recursive Utility with Habits

Generating sufficiently large risk premia amid realistic fluctuations in consumption growth is a central challenge in asset pricing theory. To address this challenge, the literature frequently employs either habit formation (Abel, 1990; Jermann, 1998) or recursive preferences (Weil, 1989; Epstein and Zin, 1989; Weil, 1989). In this paper, our methodology allows us to determine from a statistical standpoint which formulation of the utility function is the most likely to have generated the sample. Relative to our baseline, imposing $\theta = 1 - \sigma$ eliminates the distinction between IES and risk aversion, while imposing $\varpi = 1$ shuts down effects from consumption habits. Removing both effects collapses

[41] Other estimated parameters for alternative utilities are provided in section D.4 of the online appendix

our function to the standard case of expected utility. Table VI presents four alternative preference specifications, ranging from the most sophisticated to the simplest: recursive preferences with habit formation (EZWH), recursive preferences (EZW), habit formation (Habits), and CRRA. Note that the latter represents the utility model most commonly used in macroeconomics and integrated assessment models.

Table VI: Comparison of prior and posterior model probabilities under alternative preferences

In the table, *EZWH* denotes recursive utility with lifestyle habits; *EZW* denotes recursive utility without habit formation ($\varpi = 1$); *Habit* denotes utility with habit formation ($\theta = 1-\sigma$); *CRRA* in the final column corresponds to neither habits nor separation between IES and risk aversion. The prior probability denotes the degree of belief in the model's validity before observing the data and is assumed to be the same across the four competing models. The marginal data density is calculated by integrating the likelihood of the observed data over all possible values of the model parameters, weighted by the prior distribution of these parameters. The Bayes ratio is the ratio of the marginal data densities of two models, using the EZWH specification as the reference. The posterior model probability reflects how the observed data update the initial beliefs (priors) about the models. The model with the highest posterior probability (or Bayes ratio) is typically considered the most likely to be the correct model given the data. The posterior probabilities of all models under consideration usually sum to 1. The prediction errors (RMSE) indicate the in-sample root mean squared errors of prediction at forecasting horizons of one to four quarters for each observable variable. Lower RMSE values suggest better model performance.

	EZWH	EZW	Habits	CRRA
Restriction on habits:	–	$\varpi = 1$	–	$\varpi = 1$
Restriction on recursive preferences:	–	–	$\theta = 1-\sigma$	$\theta = 1-\sigma$
Posterior comparison				
Prior probability	0.25	0.25	0.25	0.25
Log marginal data density	−1 788.27	−1 824.62	−1 789.88	−1 847.20
Bayes ratio	1.00	0.00	0.347	0.00
Posterior model probability	0.74	0.00	0.26	0.00
Prediction errors (RMSE)				
Output growth	1.108	1.109	1.122	1.149
Consumption-to-GDP	0.618	0.605	0.607	0.768
Emission growth	2.460	2.502	2.465	2.445
Risk-free rate	0.618	0.706	0.662	0.840
Equity premium	8.272	8.226	8.210	8.598

For a given sample, we examine which of the four different data generating processes (DGP) is most likely to have generated the sample. The DGP exhibiting the highest log marginal data density is EZWH. To facilitate the interpretation of log marginal data density, model comparison can be expressed as probabilities. First, we impose an uninformative prior distribution over each model (i.e., 25% prior probability). Table VI displays both the posterior odds ratios and model probabilities, using EZWH as the benchmark model. The prior probability of 0.25 is significantly updated by the data. The EZWH model has a high posterior probability of 0.74, indicating that given the data, this model is the most likely among the ones considered. The Habits model has a posterior probability of 0.26. Although the data do not favor it as strongly as the EZWH model, it is still a contender, unlike the EZW and CRRA models.

To understand why one model performs better than another, we examine the in-sample prediction errors for each observable variable. We calculate the in-sample root mean squared errors (RMSE) for predictions at forecasting horizons of one to four quarters.[42] The EZWH model generally fits well across all metrics, achieving the lowest RMSE for output growth and the risk-free rate, and it performs well on other metrics too, making it a robust overall model. The EZW model performs slightly better for the consumption-to-GDP ratio and the equity premium, but not by a significant margin. However, it underperforms with respect to the risk-free rate, which is not surprising given that Bansal and Yaron (2004)'s recursive utility model is known for not generating sufficiently large variations in the risk-free rate. The Habits model shows comparable performance but does not consistently outperform the EZWH model. The CRRA model generally exhibits the highest RMSE values, indicating the poorest fit among the models.

From this section, we can conclude that a core ingredient in replicating US financial and macroeconomic data is habit formation, as it underpins the two best predictive models. Much of this performance is driven by a lower RMSE on the risk-free rate. As highlighted in the SMM section, this finding carries significant implications, suggesting substantial time variation in the SDF, a highly procyclical carbon tax, and a reduction in risk premia.

4.4 Predictability of Asset Returns

Thus far, our analysis has focused primarily on unconditional moments. In this section, we extend the evaluation to conditional moments, specifically assessing the predictability of asset returns. Since the work of Fama and French (1989), many studies have shown that conditional expected excess returns are inversely related to business conditions. Does our framework capture asset return predictability? One simple way to assess predictability both in the data and in the model is to run a local projection for a horizon $h = \{0, 1, \dots, 8\}$:

$$\Delta x_{t+h}^{i} = \alpha(h) + \gamma_t(h) + \beta(h) \log\left(P_{Et}^{i}/D_t^{i}\right) + \varepsilon_t, \tag{27}$$

[42] A model with smaller RMSE values demonstrates relatively better forecasting performance, which correlates with a higher marginal data density. This analysis allows us to identify the specific observed variables on which a model performs better, as indicated by its lower RMSE.

where Δx^i_{t+h} is the cumulative change in the target financial return from t to $t+h$, P^i_{Et}/D^i_t is the price-dividend ratio, $\alpha(h)$ is an intercept, $\gamma_t(h)$ is a time fixed effect, and $\beta(h)$ represents asset return predictability at horizon h. Because the dependent variable Δx^i_{t+h} is expressed in difference from a benchmark, each estimated $\beta(h)$ is in cumulative effects from t to $t+h$.

Figure 1: Predictability of asset returns, models vs. data

The gray areas are obtained by estimating Equation 27 on the sample data (implying $i = 1$), while each line is obtained on the panel version of Equation 27. We simulate data from each model, sampling $i = 100$ chains of size 198 periods to reach an asymptotic estimation of $\beta(h)$. Because residuals exhibit serial correlation as documented by Òscar Jordà (2005), confidence intervals are built on the Newey-West heteroscedasticity and autocorrelation consistent (HAC) estimator. Data for dividends are given in appendix B.

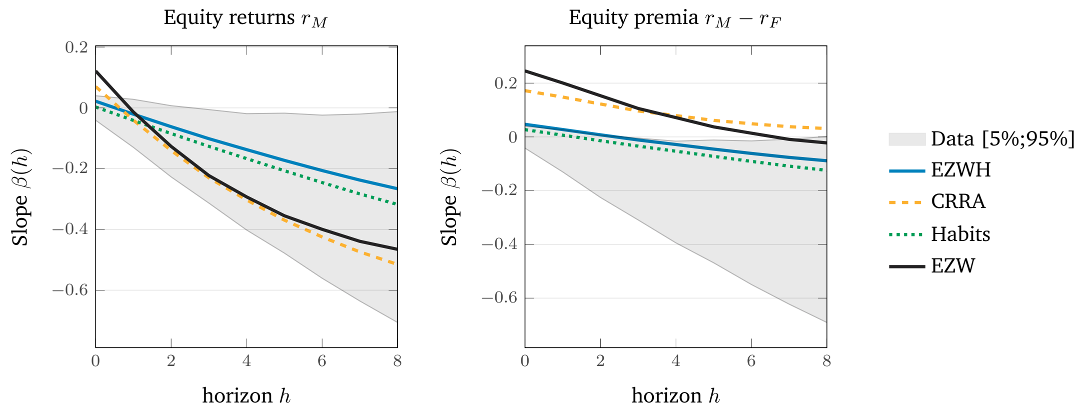


Figure 1 reports the estimates for different specifications of the utility function as well as the confidence intervals of the coefficients estimated on the data. As already documented in the literature, (i) the regression coefficients associated with cumulative returns and excess returns are negative, (ii) their absolute values increase with the horizon, and (iii) their predictability increases as the horizon increases.[43] We will next evaluate how well each model captures the observed predictability of asset returns. Notably, models with habits are able to capture the predictability of asset returns over 9 quarters, while the short-horizon predictions made by the Epstein-Zin-Weil (EZW) and power utility models fall outside the confidence intervals reported in the left panel of Figure 1.

The relation between the current price–dividend ratio and future stock returns may be due to the predictability of the risk-free rate, the equity premium, or both. As the data suggest that predictability is driven only by the equity premium, we report on the right-hand side of Figure 1 the local projection results when the equity premium is the

[43]This last pattern can be observed from the upper bound of the confidence interval going below zero for three periods and remaining there for several periods. Confidence intervals exclude zero at a 68% confidence level.

dependent variable. The results confirm that the time variation in conditional expected excess returns implies a significant degree of predictability. The gap in terms of replicating time variation in conditional equity returns is mainly driven by the equity premium: habit formation shows that it is possible to reproduce the quantitative magnitude of these coefficients when other formulations do not.

Therefore, one can conclude that models with habits also perform better in capturing the predictability of asset returns.

4.5 A Historical Path of the Optimal Carbon Price in the United States

The advantage of using Bayesian estimation is that the model can replicate the historical path of the observable variables.[44] In particular, we can answer the following question: what would the optimal carbon price v_E have been in the United States from 1973Q3 to 2023Q1 had this optimal policy been implemented? Figure 2 reports the historical time path of an optimal carbon tax for the US economy.[45]

Therefore, our optimal carbon price as reported in Figure 2 is procyclical for all types of utility considered. It is not new that optimal carbon taxes are procyclical. Because the social cost of carbon is the present value of future marginal climate damages, its procyclicality is driven by two complementary forces. The first factor is the marginal climate damages, which are proportional to output and thus inherently procyclical (see Golosov et al., 2014). The second factor is the behavior of the SDF, regardless of the preference type. When consumption is low, investors anticipate higher future consumption growth, leading to a decrease in the SDF, which reduces the weight placed on future marginal damages. As a result, the carbon price declines.

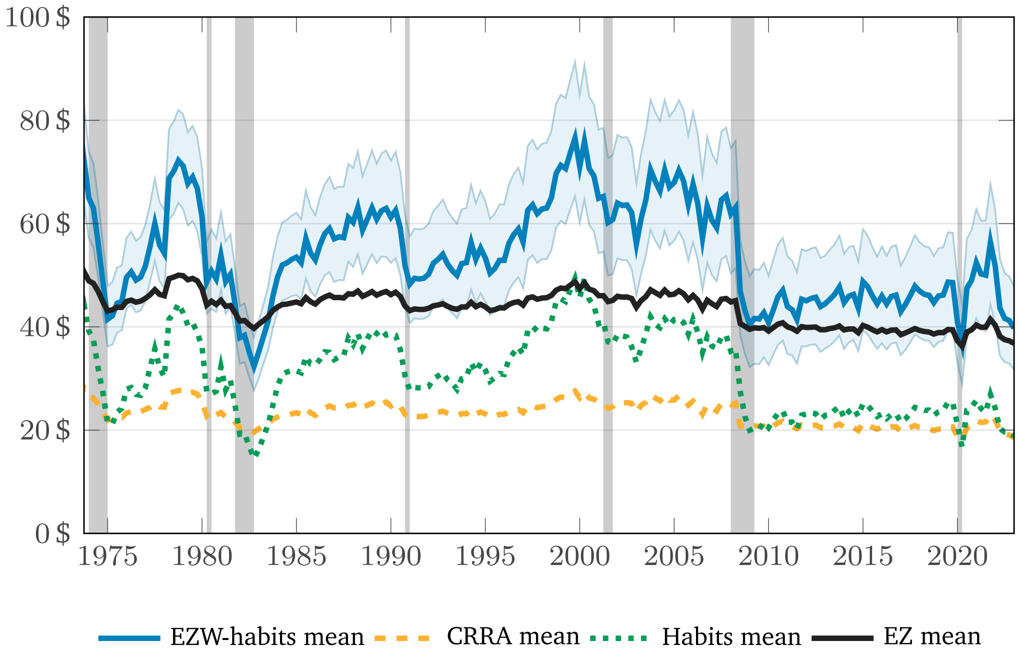
Given a power utility function as employed by Heutel (2012), Figure 2 shows that the optimal carbon tax is indeed procyclical, but less intense than under recursive utility with habits. By shutting off different mechanisms in the utility function, one can note that the procyclical nature of the social cost of carbon is shaped by consumption habits, thus corroborating the finding in the moment analysis, in section 3. Indeed, habits, as discussed by Abel (1990) and Campbell and Cochrane (1999), typically embed significant time variation in the stochastic discount factor. This utility function makes the agent more risk-averse in bad times, when consumption is low relative to its past history, than in

[44]Once the sequence of shocks has been obtained from the filtering phase of the estimation, it is then possible to simulate the model by feeding it with the sequence of shocks that replicate the sample. We can then also construct counterfactual scenarios when the calibration or equations are changed.

[45]Note that we provide a robustness check of this exercise in Section C.1 of the online appendix, based on the model estimated via SMM, fed with Solow residuals.

Figure 2: Historical path of the social cost of carbon in the United States under alternative preferences

The simulated path is expressed in levels. The blue shaded area is the parametric uncertainty at the 95% confidence level, drawn from 500 Metropolis-Hastings random iterations. The blue line represents the mean of these 500 simulated paths. The gray shaded areas are NBER-dated recessions in the U.S. The carbon tax is expressed in U.S. dollars per ton of CO_2 from the model via the expression $1,000\, v_{E,t}$. To derive the path under alternative preferences, each channel (IES/RA separation and habit formation) is deactivated, reducing the preferences to standard power utility in the simplest case.



good times, when consumption is high. The procyclical nature of the stochastic discount factor is particularly reinforced in bad times with habits, generating a large decline in the social cost of carbon during recessions.

4.6 The Amplification Channel of Fluctuating Uncertainty on Variations of the Carbon Price

Why should the carbon price be higher in a model capable of generating a substantial equity premium? While the climate risk literature has connected these aspects on artificial series, it has not evaluated this question using observed data. Our framework can be utilized to track real-time uncertainty and its quantitative effect on the optimal carbon price.

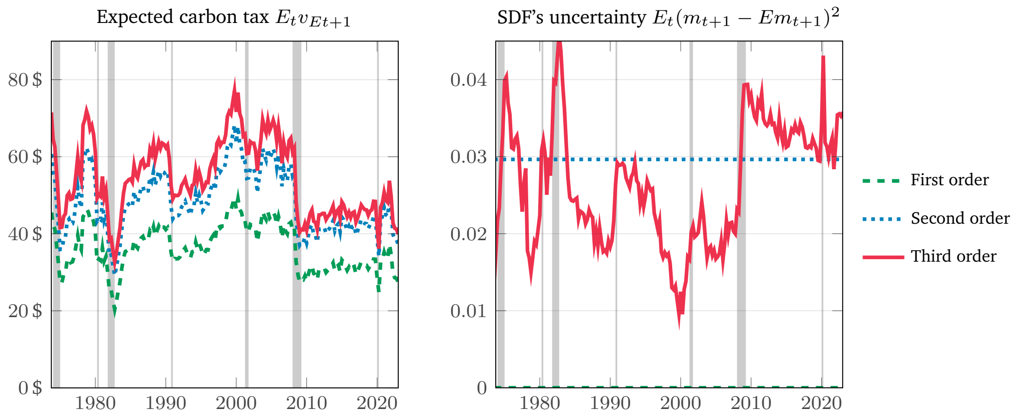
In traditional consumption-based asset pricing models, uncertainty is driven entirely by exogenous shocks, such as stochastic TFP, which cause unexpected shifts in fundamental payoffs like financial assets and consumption, thereby generating risk premia. In this section, we ask how this uncertainty also shapes the carbon price. To understand the mechanism, we build on recent work by Mumtaz and Theodoridis (2020) on uncertainty. Exploiting the recursive form of the solution of our model, the one-step-ahead expectation can be written as:[46]

$$E_t v_{Et+1} = \omega \left(H_z z_t + h_{\sigma\sigma}\sigma^2 + H_{zz}\mathrm{var}_t(z_t) + h_{\sigma\sigma\sigma}\sigma^3 + H_{zzz}\mathrm{skew}_t(z_t) \right) \tag{28}$$

where z_t is the state vector of the economy. The matrices $H_z, H_{zz}, H_{zzz}, h_{\sigma\sigma}$, and $h_{\sigma\sigma\sigma}$ denote the derivatives of the system with respect to the state and/or shock vectors for different orders and evaluated at the non-stochastic steady state, while ω is a selection matrix[47]. Thus, along with the current state, z_t, the higher moments of the system $var_t(z_t)$ and $skew_t(z_t)$ directly affect the investor's prediction about the future carbon price, and these moments are time-varying due to the specification of the agents' preferences, summarized in the SDF.

Figure 3: The role of uncertainty in shaping the price of carbon

The gray shaded areas are NBER-dated recessions in the U.S. Each simulation represents the path of the variable with the model approximated up to a specific order of the Taylor expansion.



In Figure 3, we disable or enable second- and third-order terms to evaluate how risk shapes the carbon price.[48] We measure this sensitivity to cyclical factors such as

[46]We focus on expected variables because our analysis emphasizes uncertainty, which predominantly impacts forward-looking variables. Additionally, the recursive structure of expected variables excludes the realization of shocks, making the discussion more straightforward.

[47]The term $\mathrm{var}_t(z_t) = E_t(z_t \otimes z_t)$ is the column-stacked covariance matrix of z_t at time t and $\mathrm{skew}_t(z_t) = E_t(z_t \otimes z_t \otimes z_t)$ is the column-stacked skewness matrix of z_t at time t.

[48]In Section C.1 of the online appendix, we also provide a robustness check of the role of uncertainty in the version of the model estimated via SMM, fed with Solow residuals. In Section D.5, we provide

$H_{zz}\text{var}_t(z_t)$ and $H_{zzz}\text{skew}_t(z_t)$, as well as long-term effects such as $h_{\sigma\sigma}\sigma^2$ and $h_{\sigma\sigma\sigma}\sigma^3$. The right panel of Figure 3 investigates the endogenous volatility of the SDF. Bansal and Yaron (2004) showed that fluctuations in uncertainty are crucial for capturing key features of finance data. Our framework captures significant fluctuations in uncertainty, which tends to increase during recessions.

At the first-order approximation, fluctuations in the carbon tax are procyclical, echoing the findings of Heutel (2012) and Golosov et al. (2014). In a linear economy with certainty equivalence, there is no endogenous uncertainty, causing the SDF uncertainty on the right panel of Figure 3 to be zero. However, the influence of finance, driven by EZWH preferences, becomes prominent in determining the carbon price at higher-order approximations.

A second-order approximation captures the variance, $\text{var}_t(z_t)$ and $h_{\sigma\sigma}\sigma^2$, of the model variables, causing the social cost of carbon to rise permanently. This rise is driven by a precautionary motive that increases the demand for risk-free assets to smooth consumption. Consequently, the risk-free rate permanently increases by a constant proportion of the variance of consumption, referred to as the prudence effect by Van Den Bremer and Van Der Ploeg (2021) or as precautionary savings in the asset pricing literature. By increasing the stochastic discount factor, the future marginal values receive a higher weight in present terms, increasing the SCC. At the second order, the model captures a fixed level of uncertainty, as shown by the blue dotted line in the right panel of Figure 3, primarily increasing the carbon tax through the term $h_{\sigma\sigma}\sigma^2$. This explains why the distance between the blue and green lines of the left panel remains mostly constant over the cycles.

A third-order approximation captures skewness and other higher-order moments, allowing the model to account for the time-varying nature of risk premia and the impact of uncertainty on asset and carbon prices. Uncertainty becomes time-varying, rising during expansions and decreasing during recessions, as documented by Bansal and Yaron (2004).[49] Investors under EZW dislike uncertainty, and therefore require a higher premium in compensation. Third-order terms always contribute positively to the SCC, as the solid red line never crosses the dashed blue one (indicating that $h_{\sigma\sigma\sigma}\sigma^3$ is relatively

the same figure with power utility. Under CARA, the SDF's uncertainty is negligible, making risk and the order of approximation irrelevant in determining the carbon price. Results are quantitatively and qualitatively similar under SMM or Bayesian estimations.

[49] Recall that our framework is estimated using risk premia as observable variables, which can vary only with third-order approximation. Therefore, risk fluctuations, embedded into third-order terms, reflect realistic fluctuations in uncertainty grounded by the data.

large). Time-varying skewness, $H_{zzz}\text{skew}_t(z_t)$, drastically increases the procyclical nature of the carbon tax, thus showing again how finance is important in the shaping the cyclical nature of the carbon tax.

Overall, like Folini et al. (2024); Barnett et al. (2020), we find that uncertainty increases the social cost of carbon. In a risky economy, priced accurately by a finance-consistent SDF, the price of carbon is procyclical and increases by an average of \$20 per ton of carbon, which represents a 50% increase with respect to the risk-free linear economy.

5 Conclusion

This paper contributes to the growing literature at the intersection of finance and climate economics by examining the role of empirically grounded discounting in determining the optimal carbon price. We develop a framework that integrates key elements of macro-finance with components from the integrated assessment model literature.

Our findings underscore the importance of incorporating realistic financial dynamics into climate policy models. Unlike traditional macro-finance approaches, our model includes consumption habits and is estimated on a range of asset pricing moments. This framework captures realistic fluctuations in the stochastic discount factor (SDF), which directly impacts the social cost of carbon (SCC). Specifically, our model estimates a carbon price that is 32% higher and five times more procyclical than that derived from standard power utility models. Without realistic discounting, the carbon tax would be set too low on average and would not decrease sufficiently during recessions, leading to an unnecessarily high economic burden in addressing environmental challenges.

Moreover, our analysis highlights the powerful role that the procyclicality of the carbon tax—driven by consumption habits—plays in reducing aggregate risk. This reduction in risk dampens the precautionary savings motive among investors, leading to a higher risk-free rate and lower risk premia.

6 Bibliography

A. B. Abel. Asset Prices under Habit Formation and Catching Up with the Joneses. *American Economic Review*, 80(2):38–42, May 1990. URL https://ideas.repec.org/a/aea/aecrev/v80y1990i2p38-42.html.

A. B. Abel. Risk premia and term premia in general equilibrium. *Journal of Monetary Economics*, 43(1):3–33, February 1999. URL https://ideas.repec.org/a/eee/moneco/v43y1999i1p3-33.html.

D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American Economic Review*, 102(1):131–66, 2012. URL https://EconPapers.repec.org/RePEc:cpr:ceprdp:8660.

M. Baker, D. Bergstresser, G. Serafeim, and J. Wurgler. Financing the response to climate change: The pricing and ownership of u.s. green bonds. NBER Working Papers 25194, National Bureau of Economic Research, Inc, 2018. URL https://EconPapers.repec.org/RePEc:nbr:nberwo:25194.

R. Bansal and A. Yaron. Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, 59(4):1481–1509, 2004. URL https://EconPapers.repec.org/RePEc:nbr:nberwo:8059.

R. Bansal, D. Kiku, and M. Ochoa. Climate change risk. *Mimeo, Duke University*, 2019. URL https://www.frbsf.org/economic-research/wp-content/uploads/sites/4/Paper-5-2019-11-8-Kiku-1PM-1st-paper.pdf.

M. Barnett, W. Brock, L. P. Hansen, and H. Hong. Pricing Uncertainty Induced by Climate Change. *The Review of Financial Studies*, 33(3):1024–1066, 2020. URL https://ideas.repec.org/a/oup/rfinst/v33y2020i3p1024-1066..html.

M. Barnett, W. Brock, and L. P. Hansen. Climate Change Uncertainty Spillover in the Macroeconomy. In *NBER Macroeconomics Annual 2021, volume 36*, NBER Chapters. National Bureau of Economic Research, Inc, November 2021. URL https://ideas.repec.org/h/nbr/nberch/14556.html.

L. Barrage. Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy. *The Review of Economic Studies*, 87(1):1–39, 2020. URL https://EconPapers.repec.org/RePEc:oup:restud:v:87:y:2020:i:1:p:1-39.

R. J. Barro. Rare Disasters, Asset Prices, and Welfare Costs. *American Economic Review*, 99(1):243–264, March 2009. URL https://ideas.repec.org/a/aea/aecrev/v99y2009i1p243-64.html.

S. Basu and B. Bundick. Uncertainty Shocks in a Model of Effective Demand. *Econometrica*, 85:937–958, May 2017. URL https://ideas.repec.org/a/wly/emetrp/v85y2017ip937-958.html.

Y. Cai and T. S. Lontzek. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734, 2019. URL https://EconPapers.repec.org/RePEc:arx:papers:1504.06909.

J. Y. Campbell and J. H. Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2):205–251, 1999. URL https://EconPapers.repec.org/RePEc:hrv:faseco:3119444.

G. Casey, S. Fried, and M. Gibson. Understanding climate damages: Consumption versus investment. Technical report, 2024. URL https://ideas.repec.org/p/ces/ceswps/_9499.html.

J. Chen, C. Xu, M. Gao, and D. Li. Carbon peak and its mitigation implications for china in the post-pandemic era. *Nature*, Scientific Report 12(3473), 2022. URL https://www.nature.com/articles/s41598-022-07283-4.

L. J. Christiano, R. Motto, and M. Rostagno. Risk shocks. *American Economic Review*, 104(1):27–65, 2014. URL https://EconPapers.repec.org/RePEc:nbr:nberwo:18682.

J. Cochrane. Macro-finance. *Review of Finance*, 21(3):945–985, 2017. URL https://EconPapers.repec.org/RePEc:oup:revfin:v:21:y:2017:i:3:p:945-985.

J. H. Cochrane and L. P. Hansen. Asset Pricing Explorations for Macroeconomics. In *NBER Macroeconomics Annual 1992, Volume 7*, NBER Chapters, pages 115–182. National Bureau of Economic Research, Inc, November 1992. URL https://ideas.repec.org/h/nbr/nberch/10992.html.

I. Dew-becker. Bond pricing with a time–varying price of risk in an estimated medium–scale bayesian dsge model. *Journal of Money, Credit and Banking*, 46(5):837–888, 2014. URL https://EconPapers.repec.org/RePEc:wly:jmoncb:v:46:y:2014:i:5:p:837-888.

S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019. URL https://EconPapers.repec.org/RePEc:eee:jeeman:v:96:y:2019:i:c:p:108-129.

S. Dietz, F. van der Ploeg, A. Rezai, and F. Venmans. Are economists getting climate dynamics right and does it matter? *Journal of the Association of Environmental and Resource Economists*, 8(5):895–921, 2021. URL https://EconPapers.repec.org/RePEc:ucp:jaerec:doi:10.1086/713977.

L. G. Epstein and S. E. Zin. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969, 1989. URL https://EconPapers.repec.org/RePEc:ucp:jpolec:v:99:y:1991:i:2:p:263-86.

R. Fair and J. Taylor. Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models. *Econometrica*, 51:1169–1185, 1987. URL https://EconPapers.repec.org/RePEc:nbr:nberte:0005.

E. F. Fama and K. R. French. Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics*, 25(1):23–49, November 1989. URL https://ideas.repec.org/a/eee/jfinec/v25y1989i1p23-49.html.

D. Folini, A. Friedl, F. Kübler, and S. Scheidegger. The climate in climate economics. *Review of Economic Studies*, page rdae011, 2024. URL https://EconPapers.repec.org/RePEc:arx:papers:2107.06162.

J. C. Fuhrer. Habit formation in consumption and its implications for monetary-policy models. *American Economic Review*, 90(3):367–390, June 2000. doi: 10.1257/aer.90.3.367. URL https://www.aeaweb.org/articles?id=10.1257/aer.90.3.367.

S. Giglio, B. T. Kelly, and J. Stroebel. Climate Finance. NBER Working Papers 28226, National Bureau of Economic Research, Inc, Dec. 2020. URL https://ideas.repec.org/p/nbr/nberwo/28226.html.

M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014. URL https://EconPapers.repec.org/RePEc:cpr:ceprdp:8527.

L. Hansen and K. Singleton. Generalized instrumental variables estimation of nonlinear rational expectations models. *Econometrica*, 50(5):1269–86, 1982. URL https://EconPapers.repec.org/RePEc:ecm:emetrp:v:50:y:1982:i:5:p:1269-86.

G. Heutel. How should environmental policy respond to business cycles? Optimal policy under persistent productivity shocks. *Review of Economic Dynamics*, 15(2):244–264, 2012. URL https://EconPapers.repec.org/RePEc:red:issued:10-62.

F. Ita and M. Robert. The Economics of Controlling Stock Pollutants: An Efficient Strategy for Greenhouse Gases. *Journal of Environmental Economics and Management*, 25(1):76–88, July 1993. URL https://ideas.repec.org/a/eee/jeeman/v25y1993i1p76-88.html.

I. Jaccard. Asset returns and labor supply in a production economy. *Journal of Money, Credit and Banking*, 46(5):889–919, 2014. ISSN 00222879, 15384616. URL http://www.jstor.org/stable/42920154.

I. Jaccard. Monetary Asymmetries Without (And With) Price Stickiness. *International Economic Review*, 65(2):1003–1047, May 2024. doi: 10.1111/iere.12677. URL https://ideas.repec.org/a/wly/iecrev/v65y2024i2p1003-1047.html.

U. Jermann and V. Quadrini. Macroeconomic effects of financial shocks. *American Economic Review*, 102(1):238–71, 2012. URL https://EconPapers.repec.org/RePEc:aea:aecrev:v:102:y:2012:i:1:p:238-71.

U. J. Jermann. Asset pricing in production economies. *Journal of Monetary Economics*, 41(2):257–275, 1998. URL https://EconPapers.repec.org/RePEc:eee:moneco:v:76:y:2015:i:c:p:87-106.

G. Kaltenbrunner and L. A. Lochstoer. Long-Run Risk through Consumption Smoothing. *The Review of Financial Studies*, 23(8):3190–3224, 05 2010. ISSN 0893-9454. doi: 10.1093/rfs/hhq033. URL https://doi.org/10.1093/rfs/hhq033.

G. Kaplan, B. Moll, and G. L. Violante. Monetary Policy According to HANK. *American Economic Review*, 108(3):697–743, March 2018. URL https://ideas.repec.org/a/aea/aecrev/v108y2018i3p697-743.html.

R. G. King, C. I. Plosser, and S. T. Rebelo. Production, growth and business cycles: I. the basic neoclassical model. *Journal of Monetary Economics*, 21(2):195–232, 1988.

ISSN 0304-3932. doi: https://doi.org/10.1016/0304-3932(88)90030-X. URL https://www.sciencedirect.com/science/article/pii/030439328890030X.

N. R. Kocherlakota. On the 'discount' factor in growth economies. *Journal of Monetary Economics*, 25(1):43–47, 1990. ISSN 0304-3932. doi: https://doi.org/10.1016/0304-3932(90)90043-4. URL https://www.sciencedirect.com/science/article/pii/0304393290900434.

L. Kogan, J. Li, and H. H. Zhang. Operating hedge and gross profitability premium. *The Journal of Finance*, 78(6):3387–3422, 2023. URL https://ideas.repec.org/p/nbr/nberwo/30241.html.

R. Kollmann. Tractable likelihood-based estimation of non-linear DSGE models. *Economics Letters*, 161:90–92, 2017. URL https://EconPapers.repec.org/RePEc:eee:ecolet:v:161:y:2017:i:c:p:90-92.

H. Kung and L. Schmid. Innovation, Growth, and Asset Prices. *Journal of Finance*, 70(3):1001–1037, June 2015. URL https://ideas.repec.org/a/bla/jfinan/v70y2015i3p1001-1037.html.

R. Mehra and E. C. Prescott. The equity premium: A puzzle. *Journal of Monetary Economics*, 15(2):145–161, March 1985. URL https://ideas.repec.org/a/eee/moneco/v15y1985i2p145-161.html.

H. Mumtaz and K. Theodoridis. Dynamic effects of monetary policy shocks on macroeconomic volatility. *Journal of Monetary Economics*, 114(C):262–282, 2020. doi: 10.1016/j.jmoneco.2019.03. URL https://ideas.repec.org/a/eee/moneco/v114y2020icp262-282.html.

I. B. Nath, V. A. Ramey, and P. J. Klenow. How much will global warming cool global growth? Technical report, National Bureau of Economic Research, 2024.

W. Nordhaus. *A Question of Balance: Weighing the Options on Global Warming Policies*. Yale University Press, 2008. ISBN 9780300137484. URL http://www.jstor.org/stable/j.ctt1npzkh.

W. D. Nordhaus. To slow or not to slow: the economics of the greenhouse effect. *Economic Journal*, 101(407):920–937, 1991. URL https://EconPapers.repec.org/RePEc:ecj:econjl:v:101:y:1991:i:407:p:920-37.

W. D. Nordhaus. Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7):1518–1523, 2017. URL https://www.pnas.org/doi/10.1073/pnas.1609244114.

L. Pàstor, R. F. Stambaugh, and L. Taylor. Dissecting Green Returns. CEPR Discussion Papers 16260, C.E.P.R. Discussion Papers, June 2021. URL https://ideas.repec.org/p/cpr/ceprdp/16260.html.

G. D. Rudebusch and E. T. Swanson. The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks. *American Economic Journal: Macroeconomics*, 4(1):105–143, January 2012. URL https://ideas.repec.org/a/aea/aejmac/v4y2012i1p105-43.html.

F. Smets and R. Wouters. Shocks and frictions in US business cycles: A bayesian DSGE approach. *American Economic Review*, 97(3):586–606, 2007. URL https://EconPapers.repec.org/RePEc:ecb:ecbwps:2007722.

N. L. Stokey. Are there limits to growth? *International Economic Review*, pages 1–31, 1998. URL https://EconPapers.repec.org/RePEc:ier:iecrev:v:39:y:1998:i:1:p:1-31.

T. D. Tallarini. Risk-sensitive real business cycles. *Journal of Monetary Economics*, 45(3):507–532, June 2000. URL https://ideas.repec.org/a/eee/moneco/v45y2000i3p507-532.html.

C. P. Traeger. Uncertainty in the analytic climate economy. 2021.

T. S. Van Den Bremer and F. Van Der Ploeg. The risk-adjusted carbon price. *American Economic Review*, 111(9):2782–2810, September 2021. doi: 10.1257/aer.20180517. URL https://www.aeaweb.org/articles?id=10.1257/aer.20180517.

R. Van Der Ploeg. Macro-financial implications of climate change and the carbon transition. mimeo, ECB Forum on Central Banking, 2020. URL https://www.ecb.europa.eu/pub/conferences/shared/pdf/20201111_ECB_Forum/academic_paper_vanderPloeg.pdf.

R. Van Der Ploeg, C. Hambel, and H. Kraft. Asset Pricing and Decarbonization: Diversification versus Climate Action. Economics Series Working Papers 901, University of Oxford, Department of Economics, Feb. 2020. URL https://EconPapers.repec.org/RePEc:oxf:wpaper:901.

P. Weil. The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24(3):401–421, 1989. URL https://EconPapers.repec.org/RePEc:spo:wpmain:info:hdl:2441/8686.

P. Weil. Nonexpected utility in macroeconomics. *Quarterly Journal of Economics*, 105(1):29–42, 1990. URL https://EconPapers.repec.org/RePEc:hal:journl:hal-03393362.

O. D. Zerbib. The effect of pro-environmental preferences on bond prices: Evidence from green bonds. *Journal of Banking & Finance*, 98:39–60, 2019. URL https://EconPapers.repec.org/RePEc:eee:jbfina:v:98:y:2019:i:c:p:39-60.

Òscar Jordà. Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review*, 95(1):161–182, March 2005. URL https://ideas.repec.org/a/aea/aecrev/v95y2005i1p161-182.html.

A Equilibrium conditions

In this section, we collect all the equation that determine the symmetric equilibrium in our economy. Preferences are given by:

$$m_{t+1} = \beta E_t \left(U_{t+1} / E_t \{ U_{t+1}^{1-\sigma} \}^{\frac{1}{1-\sigma}} \right)^{1-\theta-\sigma} \left(\frac{x_t^{\nu}}{x_{t+1}^{\nu}} \right) \frac{\left[(1-\beta) S_{t+1}^{\theta-1} - (1-\varpi) \varphi_{t+1} \right]}{\left[(1-\beta) S_t^{\theta-1} - (1-\varpi) \varphi_t \right]} \tag{29}$$

$$\varphi_t = \beta E_t \left(U_{t+1} / E_t \{ U_{t+1}^{1-\sigma} \}^{\frac{1}{1-\sigma}} \right)^{1-\theta-\sigma} \left[(1-\beta) S_{t+1}^{\theta-1} + \varpi \varphi_{t+1} \right] \tag{30}$$

$$U_t = \left[(1-\beta) S_t^{\theta} + E_t \{ U_{t+1}^{1-\sigma} \}^{\theta/(1-\sigma)} \right]^{1/\theta} \tag{31}$$

$$Z_{t+1} = m Z_t + (1-m) \varepsilon_{B,t} C_t x_t^{-\nu} \tag{32}$$

where $S_t = \varepsilon_{B,t} C_t x_t^{-\nu} - Z_t$.

The production side is given by:

$$Y_t = \exp(-\zeta x_t) A \varepsilon_{A,t} (K_{Y,t})^{\alpha} (n_Y H_t)^{1-\alpha} \tag{33}$$

$$H_{t+1} = H_t + \kappa \exp(-\zeta x_t) \varepsilon_{A,t} (K_t - K_{Yt})^{\xi} (n_H H_t)^{1-\xi} \tag{34}$$

$$K_{t+1} = (1-\delta_K) K_t + \left(\frac{\chi_1}{1-\epsilon} \left(\frac{I_{Kt}}{K_t} \right)^{1-\epsilon} + \chi_2 \right) K_t \tag{35}$$

$$Y_t = C_t + I_{Kt} + G \varepsilon_{G,t} + \eta_1 \mu_t^{\eta_2} Y_t \tag{36}$$

$$1 = q_t \chi_1 (I_{Kt}/K_t)^{-\epsilon} \tag{37}$$

$$q_t = E_t m_{t+1} \frac{\varepsilon_{D,t+1}}{\varepsilon_{D,t}} \left(q_{t+1} \left[(1-\delta_K) + \frac{\chi_1 \epsilon}{1-\epsilon} \left(\frac{I_{t+1}}{K_{t+1}} \right)^{1-\epsilon} + \chi_2 \right] + \varrho_{t+1} \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \tag{38}$$

$$\phi_t = E_t m_{t+1} \frac{\varepsilon_{D,t+1}}{\varepsilon_{D,t}} \left[\phi_{t+1} \left(1 + (1-\xi) \frac{\Delta H_{t+1}}{H_{t+1}} \right) + \varrho_{t+1} (1-\alpha) \frac{Y_{t+1}}{H_{t+1}} \right] \tag{39}$$

$$\frac{\phi_t}{\varrho_t} = \frac{\alpha}{\xi} \frac{Y_t}{K_{Yt}} \left(\frac{\Delta H_{t+1}}{K_t - K_{Yt}} \right)^{-1} \tag{40}$$

$$\varrho_t = 1 - \eta_1 \mu_t^{\eta_2} - \frac{V_{Et}}{H_t} \iota_E (1-\mu_t) \varepsilon_{E,t} \tag{41}$$

$$V_{E,t} = \frac{\eta_2 \eta_1}{\iota_E} \mu_t^{\eta_2 - 1} \tag{42}$$

$$D_t = \left(Y_t - (1-\alpha) Y_t - I_{Kt} - \eta_1 \mu_t^{\eta_2} Y_t - e_t \iota_E V_{Et} \right) \varepsilon_{D,t} \tag{43}$$

The climate block reads as:

$$x_{t+1} = (1-\delta_X)\, x_t + \iota_X\,(e_t + e^*) \tag{44}$$

$$e_t = (1-\mu_t)\,\iota_E \frac{Y_t}{H_t} \varepsilon_{E,t} \tag{45}$$

Finally, the optimal carbon tax (in trillions per Gt CO_2) is given by

$$V_{Et} = E_t m_{t+1} \left(\iota_X \nu \frac{C_{t+1}}{x_{t+1}} + \iota_X \zeta \left(\varrho_{t+1} Y_{t+1} + \kappa \phi_{t+1} I_{Ht+1} \right) + (1-\delta_X)\, V_{Et+1} \right)$$

while exogenous shocks are given by $\ln \varepsilon_{j,t} = \rho_j \ln \varepsilon_{j,t-1} + \eta_{j,t}$ with $j = \{A, B, G, E, D\}$ and $\eta_{j,t} \sim \mathcal{N}\left(0, \operatorname{std}(\eta_j)^2\right)$, and usual asset pricing formula: $P_{Et} = E_t m_{t+1} \left(P_{Et+1} + D_{t+1}\right)$, $r_{Ft} = 1/E_t m_{t+1}$ and $r_{Mt+1} = \left(P_{Et+1} + D_{t+1}\right)/P_{Et}$.

B Data

Our data spans the period from 1973Q3 to 2023Q1, with the starting date limited by the availability of CO_2 emissions data beginning in 1973. The data for consumption, production, investment, and R&D investment are taken from the Bureau of Economic Analysis, seasonally adjusted in chained 2017 USD. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. Carbon emissions are measured from energy consumption from U.S. Energy Information Administration. The price of shares is measured by the S&P Composite Total Return Index, and dividends are measured by the S&P 500 Dividend per share, both sourced from Standard and Poor's. The risk-free rate is measured by 3-month T-bills on the secondary market from the Federal Reserve Board/Bureau of Labor Statistics.

Regarding transformations, nominal variables are converted to real terms using the consumer price index for all items from the Bureau of Economic Analysis. Data exhibiting a trend are expressed in growth rates through their logarithmic variations. The ex-post equity premium is measured by the difference between the realized log variations of the S&P index minus the risk-free rate on T-bills. The risk-free rate is deflated by subtracting expected inflation. The price-dividend ratio is measured by the ratio between the S&P 500 price index and S&P 500 dividends. The ratio of R&D is measured by R&D investment spending over private fixed investment.

Internet Appendix

(not for publication)

A Deriving the Model

A.1 The Laissez-faire or Competitive Equilibrium

We start by characterizing the laissez-faire or competitive equilibrium in the growing economy. We next derive the stationary equilibrium and then turn to the optimal allocation.

A.1.1 The Growing Economy

Stationary variables are denoted with small letters, whereas capital letters are used to denote variables exhibiting a time trend.

Households Maximisation problem of households:

$$U_t = \left[(1-\beta) \left(\frac{C_t}{x_t^{\nu}} - Z_t \right)^{\theta} + \beta \left[E_t \left(U_{t+1}^{1-\sigma} \right) \right]^{\frac{\theta}{1-\sigma}} \right]^{\frac{1}{\theta}}$$

such that:

$$T_t + W_t n_{Yt} + s_t D_t + B_t \geq C_t + P_{Et}(s_{t+1} - s_t) + p_{Bt} B_{t+1}$$

and:

$$Z_{t+1} \geq \varpi Z_t + (1-\varpi) \frac{C_t}{x_t^{\nu}}$$

Dynamic programming problem:

$$\begin{aligned} \max U_t = & \left\{ \left[(1-\beta) \left(\frac{C_t}{x_t^{\nu}} - Z_t \right)^{\theta} + \beta \left[E_t \left(U_{t+1}^{1-\sigma} \right) \right]^{\frac{\theta}{1-\sigma}} \right]^{\frac{1}{\theta}} \right. \\ & + \lambda_t \left[T_t + W_t n_{Yt} + s_t D_t + B_t - C_t - P_{Et}(s_{t+1} - s_t) - p_{Bt} B_{t+1} \right] \\ & \left. + \varphi_t \left[Z_{t+1} - \varpi Z_t - (1-\varpi) \frac{C_t}{x_t^{\nu}} \right] \right\} \end{aligned}$$

First-order conditions:

$$\lambda_t = \frac{1}{x_t^\nu}\left[U_t^{1-\theta}(1-\beta)\left(\frac{C_t}{x_t^\nu}-Z_t\right)^{\theta-1}-\varphi_t(1-\varpi)\right]$$

$$\varphi_t = U_t^{1-\theta}\beta\left[E_t\left(U_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}-1}E_t\left\{\left[U_{t+1}^{-\sigma}\frac{1}{x_{t+1}^\nu}\left[(1-\beta)U_{t+1}^{1-\theta}\left(\frac{C_{t+1}}{x_{t+1}^\nu}-Z_{t+1}\right)^{\theta-1}+\varphi_{t+1}\varpi\right]\right]\right\}$$

$$P_{Et} = U_t^{1-\theta}\beta\left[E_t\left(U_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}-1}E_t\left[U_{t+1}^{-\sigma}\frac{\lambda_{t+1}}{\lambda_t}\left(P_{Et+1}+D_{t+1}\right)\right]$$

$$p_{Bt} = \beta U_t^{1-\theta}\left[E_t\left(U_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}-1}E_t\left[U_{t+1}^{-\sigma}\frac{\lambda_{t+1}}{\lambda_t}\right]$$

Stochastic discount factor:

$$m_{t+1} = \beta U_t^{1-\theta}\left[E_t\left(U_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}-1}\left[U_{t+1}^{-\sigma}\left(\frac{x_{t+1}^\nu}{x_t^\nu}\right)^{-\nu}\frac{\left[U_{t+1}^{1-\theta}(1-\beta)\left(\frac{C_{t+1}}{x_{t+1}^\nu}-Z_{t+1}\right)^{\theta-1}-\varphi_{t+1}(1-\varpi)\right]}{\left[U_t^{1-\theta}(1-\beta)\left(\frac{C_t}{x_t^\nu}-Z_t\right)^{\theta-1}-\varphi_t(1-\varpi)\right]}\right]$$

Final good producers Maximise dividends:

$$D_t = Y_t - W_t n_{Yt} - I_{Kt} - T_{Et}e_t - \eta_1\mu_t^{\eta_2}Y_t$$

such that:

$$K_{t+1} \leq (1-\delta_K)K_t + \left(\frac{\chi_1}{1-\epsilon}\left(\frac{I_{Kt}}{K_t}\right)^{1-\epsilon}+\chi_2\right)K_t$$

$$H_{t+1} \leq H_t + I_{Ht}$$

$$Y_t \leq \exp(-\zeta x_t)\varepsilon_{At}\left(K_t - K_{Ht}\right)^\alpha\left(H_t n_{Yt}\right)^{1-\alpha}$$

$$I_{Ht} \leq \exp(-\zeta x_t)\kappa\varepsilon_{At}K_{Ht}^\xi\left(H_t n_{Ht}\right)^{1-\xi}$$

The dynamic programming problem:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} m_t \left\{ \begin{array}{c} Y_t - W_t n_{Yt} - I_{Kt} - T_{Et} e_t - \eta_1 \mu_t^{\eta_2} Y_t \\ +q_t \left[(1-\delta) K_t + \left(\frac{\chi_1}{1-\epsilon} \left(\frac{I_{Kt}}{K_t} \right)^{1-\epsilon} + \chi_2 \right) K_t - K_{t+1} \right] \\ +\phi_t \left[H_t + \exp(-\zeta x_t) \kappa \varepsilon_{At} K_{Ht}^{\xi} \left(H_t n_{Ht} \right)^{1-\xi} - H_{t+1} \right] \\ +\varrho_t \left[\exp(-\zeta x_t) \varepsilon_{At} \left(K_t - K_{Ht} \right)^{\alpha} \left(H_t n_{Yt} \right)^{1-\alpha} - Y_t \right] \\ +V_{Et} \left[e_t - (1-\mu_t) \iota_E \frac{Y_t}{\Upsilon_t} \right] \end{array} \right.$$

First-order conditions:

$$T_{Et} = V_{Et}$$

$$\eta_2 \eta_1 \mu_t^{\eta_2 - 1} = V_{Et} \iota_E \frac{1}{\Upsilon_t}$$

$$\phi_t \xi \frac{I_{Ht}}{K_{Ht}} = \varrho_t \alpha \frac{Y_t}{K_{Yt}}$$

$$q_t = E_t m_{t+1} q_{t+1} \left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\frac{I_{Kt+1}}{K_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\frac{I_{Kt+1}}{K_{t+1}} \right)^{1-\epsilon} \right] + E_t m_{t+1} \varrho_{t+1} \alpha \frac{Y_{t+1}}{K_{Yt+1}}$$

$$\phi_t = E_t m_{t+1} \phi_{t+1} \left[1 + (1-\xi) \frac{I_{Ht+1}}{H_{t+1}} \right] + E_t m_{t+1} \varrho_{t+1} (1-\alpha) \frac{Y_{t+1}}{H_{t+1}}$$

$$\varrho_t = 1$$

$$W_t = \varrho_t (1-\alpha) \frac{Y_t}{n_{Yt}}$$

$$1 = q_t \chi_1 \left(\frac{I_{Kt}}{K_t} \right)^{-\epsilon}$$

Environmental externality

$$x_{t+1} = (1-\delta_X) x_t + \iota_X \left(e_t + e_t^* \right)$$

$$e_t = (1-\mu_t) \iota_E \frac{Y_t}{\Upsilon_t}$$

A.2 The Stationary Economy

Given the law of motion (3), and subject to the restrictions on preferences discussed by King et al. (1988), it is possible to derive a stationary equilibrium where all variables are growing at the same rate. In line with established methods in the field, we achieve a stationary equilibrium by normalizing trended variables with the rate of labor augmenting technological progress, denoted as H_t.

$$\frac{H_{t+1}}{H_t} = 1 + \exp(-\zeta x_t)\kappa\varepsilon_{At}k_{Ht}^{\xi}n_{Ht}^{1-\xi}$$

where:

$$\gamma_t = H_{t+1}/H_t$$

$$u_t = \left\{\left[(1-\beta)\left(\frac{c_t}{x_t^{\nu}} - z_t\right)^{\theta} + \beta\gamma_t^{\theta}\left[E_t\left(u_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}}\right]\right\}^{\frac{1}{\theta}}$$

$$\lambda_t = \frac{1}{x_t^{\nu}}\left[u_t^{1-\theta}(1-\beta)\left(\frac{c_t}{x_t^{\nu}} - z_t\right)^{\theta-1} - \varphi_t(1-\varpi)\right]$$

where λ_t and φ_t are stationary. Define: $\widetilde{\varphi}_t = \frac{\varphi_t}{u_t^{1-\theta}}$

$$\lambda_t = \frac{1}{x_t^{\nu}}u_t^{1-\theta}\left[(1-\beta)\left(\frac{c_t}{x_t^{\nu}} - z_t\right)^{\theta-1} - \widetilde{\varphi}_t(1-\varpi)\right]$$

To simpify the exposition, in the main text, the scaled multiplier is denoted by φ_t.

$$\tau_{Et} = v_{Et}$$

$$\eta_2\eta_1\mu_t^{\eta_2-1} = v_{Et}\iota_E$$

$$\varrho_t = (1-\eta_1\mu_t^{\eta_2}) - v_{Et}(1-\mu_t)\iota_E y_t$$

$$\phi_t\xi\frac{i_{Ht}}{k_{Ht}} = \varrho_t\alpha\frac{y_t}{k_{Yt}}$$

$$i_{Ht} = \exp(-\zeta x_t)\kappa\varepsilon_{At}k_{Ht}^{\xi}n_{Ht}^{1-\xi}$$

$$\phi_t = E_t m_{t+1}\phi_{t+1}\left[1 + (1-\xi)i_{Ht+1}\right] + E_t m_{t+1}\varrho_{t+1}(1-\alpha)y_{t+1}$$

$$\widetilde{\varphi}_t = \beta\gamma_t^{\theta-1}\left[E_t\left(u_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta-(1-\sigma)}{1-\sigma}} E_t\left\{\left[u_{t+1}^{1-\sigma-\theta}\left[(1-\beta)\left(\frac{c_{t+1}}{x_{t+1}^{\nu}} - z_{t+1}\right)^{\theta-1} + \widetilde{\varphi}_{t+1}\varpi\right]\right]\right\}$$

$$p_{Et} = E_t\left[\beta\left(\gamma_t\right)^{\theta-1}\frac{u_t^{1-\theta}\left[E_t\left(u_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta-(1-\sigma)}{1-\sigma}}}{u_{t+1}^{\sigma}}\frac{\lambda_{t+1}}{\lambda_t}\gamma_t\left(p_{Et+1} + d_{t+1}\right)\right]$$

$$\gamma_t z_{t+1} - \varpi z_t - (1-\varpi)\frac{c_t}{x_t^{\nu}} = 0$$

$$1 = q_t \chi_1 \left(\frac{i_{Kt}}{k_t}\right)^{-\epsilon}$$

$$q_t = E_t m_{t+1} q_{t+1}\left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon}\left(\frac{i_{Kt+1}}{k_{t+1}}\right)^{1-\epsilon} + \chi_2 - \chi_1\left(\frac{i_{Kt+1}}{k_{t+1}}\right)^{1-\epsilon}\right] + \beta E_t m_{t+1} \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{Yt+1}}$$

$$x_{t+1} = (1-\delta_X) x_t + \iota_X \left(e_t + e_t^*\right)$$

$$e_t = (1-\mu_t)\iota_E y_t$$

$$\gamma_t k_{t+1} = (1-\delta_K) k_t + \left(\frac{\chi_1}{1-\epsilon}\left(\frac{i_{Kt}}{k_t}\right)^{1-\epsilon} + \chi_2\right) k_t$$

$$y_t = \exp(-\zeta x_t)\varepsilon_{At} k_{Yt}^{\alpha} n_{Yt}^{1-\alpha}$$

$$d_t = y_t - w_t n_{Yt} - i_{Kt} - \tau_{Et} e_t - \eta_1 \mu_t^{\eta_2} y_t$$

$$w_t = (1-\alpha)\frac{y_t}{n_{Yt}}$$

where:

$$\Upsilon_t = \digamma H_t$$

Since this parameter has no effect on the model dynamics, we normalize $\digamma$ to 1.

$$y_t = c_t + i_t + g_t$$

The stochastic discount factor is:

$$m_{t+1} = \beta \gamma_t^{\theta-1}\left[\frac{u_{t+1}}{\left[E_t\left(u_{t+1}^{1-\sigma}\right)\right]^{\frac{1}{1-\sigma}}}\right]^{1-\sigma-\theta}\left(\frac{x_{t+1}}{x_t}\right)^{-\nu}\left[\frac{(1-\beta)\left(\frac{c_{t+1}}{x_{t+1}^{\nu}} - z_{t+1}\right)^{\theta-1} - \widetilde{\varphi}_{t+1}(1-\varpi)}{(1-\beta)\left(\frac{c_t}{x_t^{\nu}} - z_t\right)^{\theta-1} - \widetilde{\varphi}_t(1-\varpi)}\right]$$

The dynamic system under laissez-faire Preferences

$$u_t = \left\{\left[(1-\beta)\left(\frac{c_t}{x_t^{\nu}} - z_t\right)^{\theta} + \beta\gamma_t^{\theta}\left[E_t\left(u_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}}\right]\right\}^{\frac{1}{\theta}}$$

$$ce_t = E_t\left(u_{t+1}^{1-\sigma}\right)$$

$$\lambda_t = \frac{1}{x_t^{\nu}} u_t^{1-\theta} \left[(1-\beta) \left(\frac{c_t}{x_t^{\nu}} - z_t \right)^{\theta-1} - \widetilde{\varphi}_t (1-\varpi) \right]$$

$$\widetilde{\varphi}_t = \beta \gamma_t^{\theta-1} \left[E_t \left(u_{t+1}^{1-\sigma} \right) \right]^{\frac{\theta-(1-\sigma)}{1-\sigma}} E_t \left\{ \left[u_{t+1}^{1-\sigma-\theta} \left[(1-\beta) \left(\frac{c_{t+1}}{x_{t+1}^{\nu}} - z_{t+1} \right)^{\theta-1} + \widetilde{\varphi}_{t+1} \varpi \right] \right] \right\}$$

$$m_t = \beta \gamma_{t-1}^{\theta-1} \left[\frac{u_t}{\left[E_{t-1} \left(u_t^{1-\sigma} \right) \right]^{\frac{1}{1-\sigma}}} \right]^{1-\sigma-\theta} \left(\frac{x_t}{x_{t-1}} \right)^{-\nu} \left[\frac{(1-\beta) \left(\frac{c_t}{x_t^{\nu}} - z_t \right)^{\theta-1} - \widetilde{\varphi}_t (1-\varpi)}{(1-\beta) \left(\frac{c_{t-1}}{x_{t-1}^{\nu}} - z_{t-1} \right)^{\theta-1} - \widetilde{\varphi}_{t-1} (1-\varpi)} \right]$$

$$\gamma_t z_{t+1} - \varpi z_t - (1-\varpi) \frac{c_t}{x_t^{\nu}} = 0$$

Dividends

$$d_t = y_t - w_t n_{Yt} - i_{Kt} - \tau_{Et} e_t - \eta_1 \mu_t^{\eta_2} y_t$$

$$w_t = (1-\alpha) \frac{y_t}{n_{Yt}}$$

Endogenous growth

$$i_{Ht} = \exp(-\zeta x_t) \kappa \varepsilon_{At} k_{Ht}^{\xi} n_{Ht}^{1-\xi}$$

$$\phi_t = E_t m_{t+1} \phi_{t+1} \left[1 + (1-\xi) i_{Ht+1} \right] + E_t m_{t+1} \varrho_{t+1} (1-\alpha) y_{t+1}$$

$$\gamma_t = 1 + i_{Ht}$$

$$\phi_t \xi \frac{i_{Ht}}{k_{Ht}} = \varrho_t \alpha \frac{y_t}{k_{Yt}}$$

Production

$$1 = q_t \chi_1 \left(\frac{i_{Kt}}{k_t} \right)^{-\epsilon}$$

$$\varrho_t = (1 - \eta_1 \mu_t^{\eta_2}) - v_{Et} (1-\mu_t) \iota_E y_t$$

$$q_t = E_t m_{t+1} q_{t+1} \left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\frac{i_{Kt+1}}{k_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\frac{i_{Kt+1}}{k_{t+1}} \right)^{1-\epsilon} \right] + E_t m_{t+1} \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{Yt+1}}$$

$$\gamma_t k_{t+1} = (1-\delta_K) k_t + \left(\frac{\chi_1}{1-\epsilon} \left(\frac{i_{Kt}}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t$$

$$y_t = \exp(-\zeta x_t) \varepsilon_{At} k_{Yt}^{\alpha} n_{Yt}^{1-\alpha}$$

Emissions

$$x_{t+1} = (1-\delta_X) x_t + \iota_X \left(e_t + e_t^* \right)$$

$$e_t = (1-\mu_t) \iota_E y_t$$

Market clearing

$$y_t = c_t + i_{Kt} + g_t$$

Asset pricing implications

$$p_{Et} = E_t \left[m_{t+1} \gamma_t \left(p_{Et+1} + d_{t+1} \right) \right]$$

$$\frac{1}{1 + r_{Ft}} = \beta \gamma_t^{\theta - 1} \left[\frac{u_{t+1}}{\left[E_t \left(u_{t+1}^{1-\sigma} \right) \right]^{\frac{1}{1-\sigma}}} \right]^{1-\sigma-\theta} \left(\frac{x_{t+1}}{x_t} \right)^{-\nu} \left[\frac{(1-\beta) \left(\frac{c_{t+1}}{x_{t+1}^{\nu}} - z_{t+1} \right)^{\theta - 1} - \widetilde{\varphi}_{t+1}(1 - \varpi)}{(1-\beta) \left(\frac{c_t}{x_t^{\nu}} - z_t \right)^{\theta - 1} - \widetilde{\varphi}_t (1 - \varpi)} \right]$$

$$E_t ep_{t+1} = \gamma_t \frac{p_{Et+1} + d_{t+1}}{p_{Et}} - (1 + r_{Ft})$$

Growth rate output

$$g_{Yt} = \log (y_t) - \log (y_{t-1}) + \log \gamma_{t-1}$$

A.3 The Centralized Equilibrium

A.3.1 The Growing Economy

Planner's Problem

$$\begin{aligned}
\max U_t = \Bigg\{ & \left[(1-\beta)\left(C_t x_t^{-\nu} - Z_t\right)^{\theta} + \beta\left[E_t\left(U_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}}\right]^{\frac{1}{\theta}} \\
& + \lambda_t\left[(1-\eta_1\mu_t^{\eta_2})Y_t - C_t - I_{Kt} - G_t\right] \\
& + \varphi_t\left[Z_{t+1} - \varpi Z_t - (1-\varpi)C_t x_t^{-\nu}\right] \\
& + \lambda_t q_t\left[(1-\delta_K)K_t + \left(\frac{\chi_1}{1-\epsilon}\left(\frac{I_{Kt}}{K_t}\right)^{1-\epsilon} + \chi_2\right)K_t - K_{t+1}\right] \\
& + \lambda_t\phi_t\left[H_t + \exp(-\zeta x_t)\kappa\varepsilon_{At}K_{Ht}^{\xi}\left(H_t n_{Ht}\right)^{1-\xi} - H_{t+1}\right] \\
& + \lambda_t\varrho_t\left[\exp(-\zeta x_t)\varepsilon_{At}\left(K_t - K_{Ht}\right)^{\alpha}\left(H_t n_{Yt}\right)^{1-\alpha} - Y_t\right] \\
& + V_{Xt}\lambda_t\left[x_{t+1} - (1-\delta_X)x_t - \iota_X\left(e_t + e_t^*\right)\right] \\
& + V_{Et}\lambda_t\left[e_t - (1-\mu_t)\iota_E\frac{Y_t}{\Upsilon_t}\right]\Bigg\}
\end{aligned}$$

First-order conditions:

$$\varrho_t = (1-\eta_1\mu_t^{\eta_2}) - V_{Et}(1-\mu_t)\iota_E\frac{Y_t}{\Upsilon_t}$$

$$V_{Et} = \iota_X V_{Xt}$$

$$\eta_2\eta_1\mu_t^{\eta_2-1} = V_{Et}\iota_E\frac{1}{\Upsilon_t}$$

$$\lambda_t = \frac{1}{x_t^{\nu}}\left[U_t^{1-\theta}(1-\beta)\left(\frac{C_t}{x_t^{\nu}} - Z_t\right)^{\theta-1} - \varphi_t(1-\varpi)\right]$$

$$V_{Xt} = E_t m_{t+1}\left\{\nu\frac{C_{t+1}}{x_{t+1}} + \zeta\varrho_{t+1}Y_{t+1} + \zeta\phi_{t+1}I_{Ht+1} + V_{Xt+1}(1-\delta_X)\right\}$$

$$\varphi_t = U_t^{1-\theta}\beta\left[E_t\left(U_{t+1}^{1-\sigma}\right)\right]^{\frac{\theta}{1-\sigma}-1}E_t\left\{\left[U_{t+1}^{-\sigma}\left[(1-\beta)U_{t+1}^{1-\theta}\left(\frac{C_{t+1}}{x_{t+1}^{\nu}} - Z_{t+1}\right)^{\theta-1} + \varphi_{t+1}\varpi\right]\right]\right\}$$

$$q_t = E_t m_{t+1}\left\{q_{t+1}\left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon}\left(\frac{I_{Kt+1}}{K_{t+1}}\right)^{1-\epsilon} + \chi_2 - \chi_1\left(\frac{I_{Kt+1}}{K_{t+1}}\right)^{1-\epsilon}\right] + \varrho_{t+1}\alpha\frac{Y_{t+1}}{K_{Yt+1}}\right\}$$

$$\phi_t = E_t m_{t+1}\left\{\phi_{t+1}\left[1 + (1-\xi)\frac{I_{Ht+1}}{H_{t+1}}\right] + \varrho_{t+1}(1-\alpha)\frac{Y_{t+1}}{H_{t+1}}\right\}$$

$$\phi_t \xi \frac{I_{Ht}}{K_{Ht}} = \varrho_t \alpha \frac{Y_t}{K_{Yt}}$$

$$1 = q_t \chi_1 \left(\frac{I_{Kt}}{K_t} \right)^{-\epsilon}$$

A.3.2 The stationary system in the centralized or first-best equilibrium

Preferences

$$u_t = \left\{ \left[(1-\beta)(c_t x_t^{-\nu} - z_t)^{\theta} + \beta \gamma_t^{\theta} \left[E_t \left(u_{t+1}^{1-\sigma} \right) \right]^{\frac{\theta}{1-\sigma}} \right] \right\}^{\frac{1}{\theta}}$$

$$ce_t = E_t \left(u_{t+1}^{1-\sigma} \right)$$

$$\lambda_t = u_t^{1-\theta} x_t^{-\nu} \left[(1-\beta)(c_t - z_t)^{\theta-1} - \widetilde{\varphi}_t (1-\varpi) \right]$$

$$\widetilde{\varphi}_t = \beta \gamma_t^{\theta-1} \left[E_t \left(u_{t+1}^{1-\sigma} \right) \right]^{\frac{\theta-(1-\sigma)}{1-\sigma}} E_t \left\{ \left[u_{t+1}^{1-\sigma-\theta} \left[(1-\beta)(c_{t+1} x_{t+1}^{-\upsilon} - z_{t+1})^{\theta-1} + \widetilde{\varphi}_{t+1} \varpi \right] \right] \right\}$$

$$\gamma_t z_{t+1} - \varpi z_t - (1-\varpi) c_t x_t^{-\nu} = 0$$

Climate

$$v_{Xt} = E_t \gamma_t m_{t+1} \left\{ \nu \frac{c_{t+1}}{x_{t+1}} + \zeta \varrho_{t+1} y_{t+1} + \zeta \phi_{t+1} i_{Ht+1} + v_{Xt+1}(1-\delta_X) \right\}$$

$$v_{Et} \iota_E = \eta_2 \eta_1 \mu_t^{\eta_2 - 1}$$

$$v_{Xt} \iota_X = v_{Et}$$

$$e_t = (1-\mu_t) \iota_E y_t$$

$$x_{t+1} = (1-\delta_X) x_t + \iota_X \left(e_t + e_t^* \right)$$

Endogenous growth

$$i_{Ht} = \exp(-\zeta x_t) \kappa \varepsilon_{At} k_{Ht}^{\xi} \left(n_{Ht} \right)^{1-\xi}$$

$$\phi_t = E_t m_{t+1} \phi_{t+1} \left[1 + (1-\xi) i_{Ht+1} \right] + E_t m_{t+1} \varrho_{t+1} (1-\alpha) y_{t+1}$$

$$\gamma_t = 1 + i_{Ht}$$

$$\phi_t \xi \frac{i_{Ht}}{k_{Ht}} = \varrho_t \alpha \frac{y_t}{k_{Yt}}$$

Production

$$\varrho_t = 1 - \eta_1 \mu_t^{\eta_2} - v_{Et} (1-\mu_t) \iota_E$$

$$1 = q_t \chi_1 \left(\frac{i_{Kt}}{k_t}\right)^{-\epsilon}$$

$$q_t = E_t m_{t+1} q_{t+1} \left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon}\left(\frac{i_{Kt+1}}{k_{t+1}}\right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\frac{i_{Kt+1}}{k_{t+1}}\right)^{1-\epsilon} \right] + E_t m_{t+1} \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{Yt+1}}$$

$$\gamma_t k_{t+1} = (1-\delta_K) k_t + \left(\frac{\chi_1}{1-\epsilon}\left(\frac{i_{Kt}}{k_t}\right)^{1-\epsilon} + \chi_2 \right) k_t$$

$$y_t = \exp(-\zeta x_t) \varepsilon_{At} \left(k_t - k_{Ht}\right)^{\alpha} n_{Yt}^{1-\alpha}$$

$$k_t = k_{Yt} + k_{Ht}$$

Market clearing

$$y_t \left(1 - \eta_1 \mu_t^{\eta_2}\right) = c_t + i_{Kt} + g_t$$

A.4 Optimal Tax in the Laissez-Faire Equilibrium

In the decentralized equilibrium, in the stationary equilibrium, we have that:

$$\tau_{Et} = v_{Et}$$

$$\eta_2 \eta_1 \mu_t^{\eta_2 - 1} = v_{Et} \iota_E$$

In the absence of environmental policy, *i.e.* $\tau_{Et} = 0$, the price of CO_2 emissions v_{Et} is zero, which leads firms to choose a zero abatement policy $\mu_t = 0$. The allocation obtained by the social planner can be achieved in the decentralized equilibrium by setting:

$$\tau_{Et} = v_{Xt} \iota_X$$

where:

$$v_{Xt} = E_t \gamma_t m_{t+1} \left\{ \nu \frac{c_{t+1}}{x_{t+1}} + \zeta \varrho_{t+1} y_{t+1} + \zeta \phi_{t+1} i_{Ht+1} + v_{Xt+1}(1-\delta_X) \right\}$$

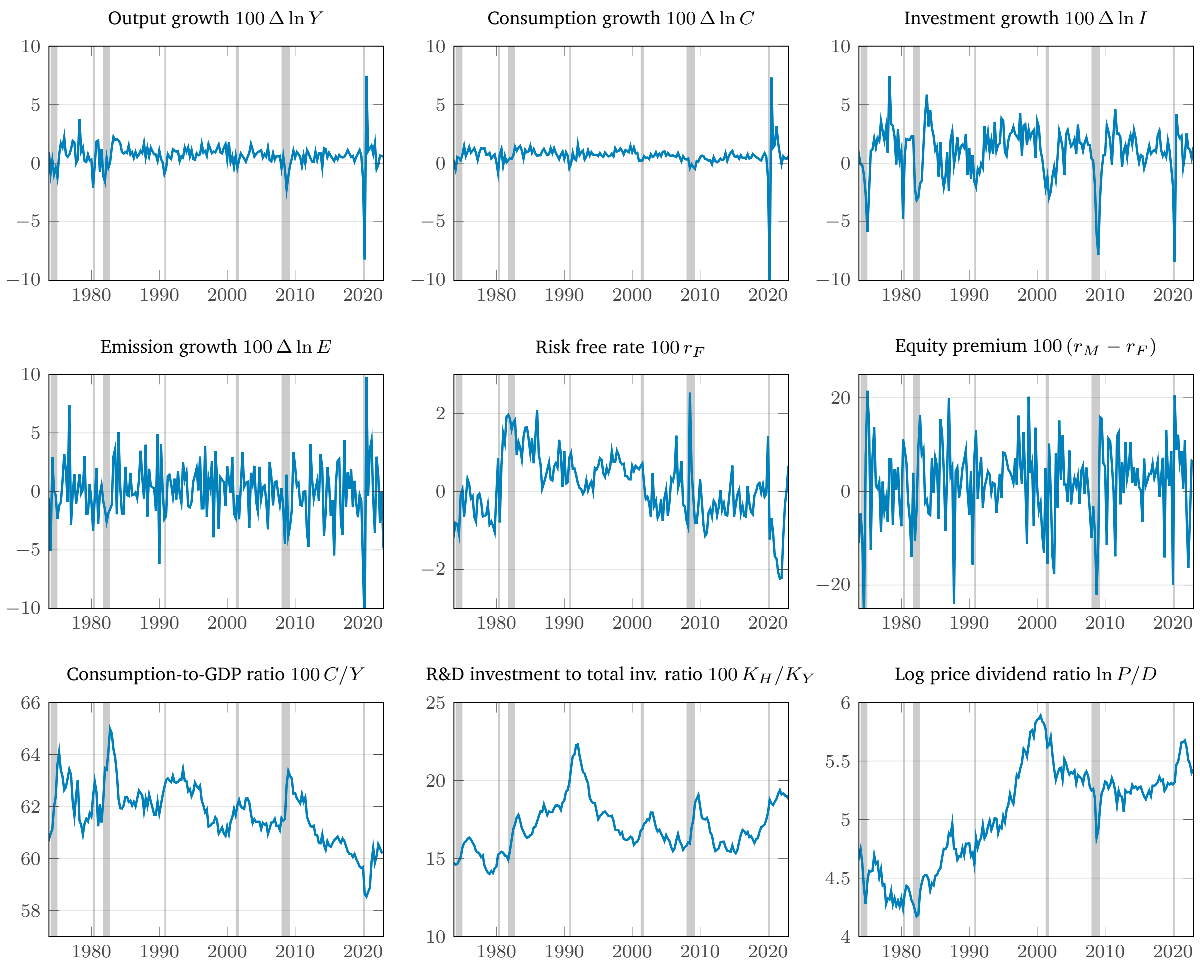
When the tax is set to its optimal level, the allocation in the laissez-faire equilibrium and in the centralized problem coincide as the set of equations characterizing the two equilibria are identical in this case. By setting the tax to its optimal level, the environmental policy can restore the first-best allocation.

B Data

B.1 Time Path of the Sample

All the data employed in the paper are reported in Figure 4, while sources are documented in Section B of the paper.

Figure 4: Transformed data used in the SMM estimation, Bayesian estimation and for return predictability



B.2 Testing the Presence of a Trend in US Carbon Dioxide Emissions Growth

To assess the presence of a deterministic trend in key macroeconomic variables, we estimated the following ordinary least squares (OLS) regression for each variable X_t:

$$\Delta \ln X_t = \alpha + \epsilon_t$$

where $\Delta \ln X_t$ represents the first difference of the natural logarithm of the variable X_t (i.e., the growth rate of X_t), α is the intercept capturing the average growth rate, and ϵ_t is the error term. The variables considered are GDP growth ($\Delta \ln Y_t$), consumption growth ($\Delta \ln C_t$), investment growth ($\Delta \ln I_t$), and emissions growth ($\Delta \ln E_t$).

The purpose of this regression is to test whether the average growth rate (α) of each variable is significantly different from zero. If α is significantly different from zero, it suggests the presence of a deterministic trend in the variable. Conversely, if α is not significantly different from zero, the variable can be considered stationary over the sample period.

Table VII: Testing the significance of a trend

This table presents the results of the OLS regression $\Delta \ln X_t = \alpha + \epsilon_t$ for GDP growth ($\Delta \ln Y_t$), consumption growth ($\Delta \ln C_t$), investment growth ($\Delta \ln I_t$), and emissions growth ($\Delta \ln E_t$). The coefficients (α), standard errors (SE), t-statistics (tStat), and p-values (pVal) are reported.

	$\hat{\alpha}$	SE	t-Stat	p-Value
GDP growth $\Delta \ln Y$	0.651	0.078	8.319	0.000
Consumption growth $\Delta \ln C$	0.643	0.072	8.932	0.000
Investment growth $\Delta \ln I$	1.014	0.151	6.718	0.000
Emissions growth $\Delta \ln e$	-0.010	0.177	-0.040	0.969

The OLS regression results in the table above indicate that the average growth rates (α) for GDP, consumption, and investment are all significantly different from zero, with p-values less than 0.01. This suggests that these variables exhibit a deterministic trend over the sample period from 1973Q3 to 2023Q1.

In contrast, emissions growth ($\Delta \ln E_t$) has a coefficient $\alpha = -0.01$ that is not significantly different from zero (t-statistic = -0.04, p-value = 0.969). This suggests that emissions growth does not exhibit a deterministic trend and can be considered stationary over the sample period.

These findings support the assumption in our model that emissions is stationary, while GDP, consumption, and investment exhibit trends.

C Additional Results to the Moment Matching Exercise

C.1 Pro-cyclicality of the Carbon Tax with Solow Residuals

To assess whether the carbon tax is pro-cyclical in the version of the economy with one exogenous source of fluctuations and parameters estimated from SMM, one can use the Solow residual approach from Jermann and Quadrini (2012). We use a production function to extract the empirical Solow residuals, $\log SR_t$, using data on output, hours worked, and investment:

$$\log SR_t = \log Y_t - \alpha \log K_t - (1 - \alpha) \log N_t, \tag{C.1}$$

A time series for $\log A_t$ is then obtained by detrending the empirical Solow residuals using a linear trend with data from 1973 to 2023 to obtain a measure that is adjusted for growth:

$$\log A_t = \log SR_t - (1 - \alpha) \log(\Gamma_t), \tag{C.2}$$

Finally, using OLS, one can capture persistence by estimating ρ_A and ε_t:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t. \tag{C.3}$$

In this expression, ε_t represents empirical white noise Solow residuals that are consistent with US data. By feeding our model estimated with SMM, we obtain Figure 5. The left panel provides the path of the carbon tax under the Solow residuals with the CARA utility function, while the right panel (Panel B) provides the path under EZWH preferences. This quantitative exercise us to provide a robustness check to two facts that we have documented in the paper.

Figure 5: Historical variations in the carbon tax simulated by the method of the Solow residual

The gray shaded areas represent NBER-dated recessions in the U.S. Each simulation represents the path of the variable with the model approximated up to a specific order of the Taylor expansion. To obtain the path under the power utility in the left panel, the IES/RA separation and habit formation are disabled.

First, one can observe the amplified pro-cyclicality of the carbon tax under realistic discounting brought by the finance part of the model, as discussed in subsection 4.5. Under power utility, the carbon tax exhibits relatively low cyclicality, remaining very stable over time but still marginally co-moving with the economic cycle. This finding aligns with the usual literature, such as Heutel (2012) and Golosov et al. (2014). In contrast, with realistic discounting, the cyclical nature of the carbon tax is exacerbated by the finance block. It increases up to $100 during the expansion driven by the Tech bubble in the 2000s but falls to $20 during the COVID-19 outbreak.

Second, as discussed in subsection 4.6, uncertainty mechanically raises the carbon tax. The Solow residual corroborates this finding. In the linear economy with certainty equivalence, there is significant procyclicality of the carbon tax under a finance-consistent utility function. The order of approximation, through precautionary saving, increases the risk-free rate and hence the present value of future damages. Consequently, this boosts the value of the carbon tax, as underlined by Folini et al. (2024) and Barnett et al. (2020). In contrast, under power utility, the order of approximation does not drastically change the price of carbon nor its cyclical nature.

C.2 Identification

To ensure robust parameter identification, we carefully explore the parameter space around the minimum point of the objective function. This approach involves examining the shape of the objective function to assess how sensitive the model is to changes in the parameters. By doing so, we can identify well-behaved regions of the parameter space and ensure that our estimates are not the result of local minima or poorly conditioned areas. This method of exploring the parameter space provides insight into the reliability of the estimated parameters and how they influence the model's ability to match the target moments. It also serves as a diagnostic tool to ensure that the model is properly specified and capable of generating realistic financial and economic dynamics.

Figure 6: Identification of the SMM model for each estimated parameter

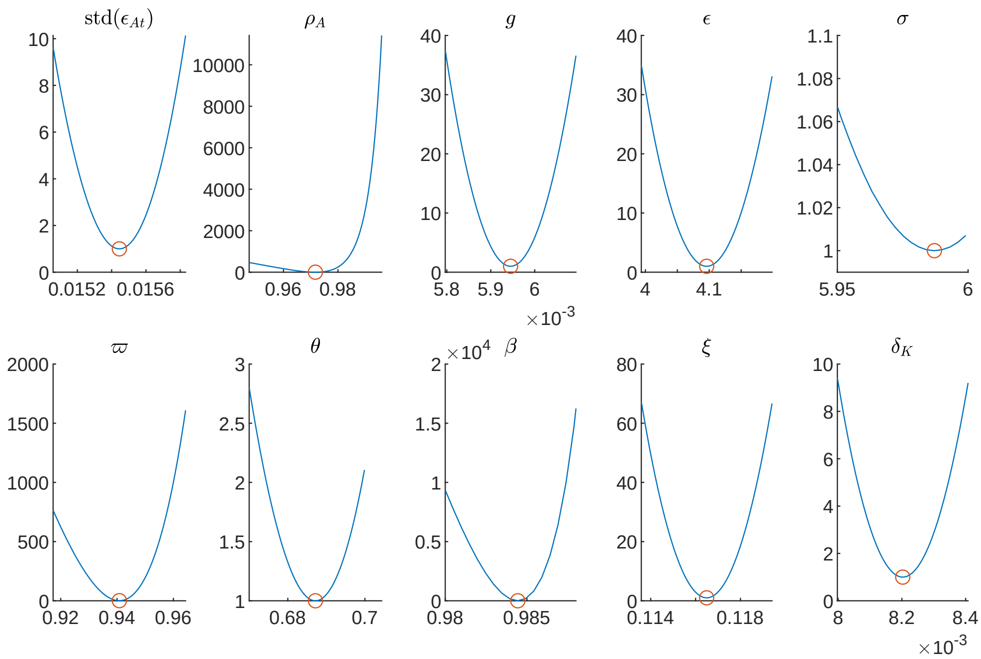


Figure 6 presents the objective function for each parameter, along with the estimated value indicated by a red dot. First, our estimates correspond to a global minimum of the objective function. By examining the changes in the objective function, it becomes clear which parameters are most crucial for matching the model to the data. The three most important parameters are the discount factor β, the persistence of productivity ρ_A, and the degree of habit formation ϖ. While habit formation proves to be highly informative, it is worth noting that the parameters related to the early resolution of uncertainty contribute much less to reducing the distance between the model and data moments.

C.3 Sensitivity analysis to carbon lifetime

Does the carbon lifetime matter in the determination of our results? The lifetime of carbon is a contentious issue in Integrated Assessment Models (IAMs), as climate models have undergone several re-assessments of the carbon cycle. These re-assessments have generally resulted in longer estimated lifetimes of carbon. While our framework for climate is necessarily simplified to obtain an analytical expression for the price of carbon, we can analyze whether our core results hold under longer carbon lifetimes, which may be more consistent with realistic carbon cycle models.

Our main calibration assumes a 120-year carbon lifetime, but we also consider scenarios with a 139-year lifetime, as in Nordhaus (1991), and even a 200-year lifetime.

Table VIII: Sensitivity of Moments under Laissez-Faire (LFP) vs. Carbon Price (CBP) Policy to Alternative Carbon Decay Rates

This table compares the moments in the Laissez-Faire (LFP) equilibrium and the economy under the optimal carbon policy (CBP) across different carbon decay rates. All moments are reported on a quarterly basis. $\hat{c}$ denotes the log deviation of consumption from the steady state, r_F the risk-free rate, ep the equity premium $(r_M - r_F)$, v_E the effective carbon tax expressed in \$ per tCO_2 (multiplied by a factor of 1,000), and $\text{cov}(\hat{v}_E, \hat{c})/\text{V}(\hat{c})$ is measured by the coefficient β estimated in the OLS regression $\hat{v}_E = \alpha + \beta\hat{c} + \epsilon_t$. Each moment is computed using 200 parallel chains with 200 observations per chain. To ensure comparability, all versions exhibit the same marginal utility loss $\nu c/x$ by adjusting ν.

	std($100\,\hat{c}$)	E($100\,r_F$)	E($100\,ep$)	E($1000\,v_E$)	std($1000\,v_E$)	$\frac{\text{cov}(\hat{v}_E,\hat{c})}{\text{V}(\hat{c})}$
	LFP [*CBP*]	LFP [*CBP*]	LFP [*CBP*]	LFP [*CBP*]	LFP [*CBP*]	LFP [*CBP*]
120 years (baseline)	4.75 [*4.48*]	0.09 [*0.35*]	1.79 [*1.53*]	0.00 [*72.3*]	0.00 [*15.7*]	0.00 [*4.95*]
139 years	4.74 [*4.45*]	0.08 [*0.34*]	1.78 [*1.53*]	0.00 [*72.8*]	0.00 [*15.8*]	0.00 [*5.02*]
200 years	4.77 [*4.38*]	0.06 [*0.33*]	1.69 [*1.51*]	0.00 [*72.0*]	0.00 [*15.6*]	0.00 [*5.25*]

As reported in the table, a longer carbon lifetime increases climate risk, as the economy will suffer longer from the presence of carbon in the atmosphere. In response, investors increase their precautionary savings motive, boosting their demand for safe assets to smooth consumption, which explains the observed decrease in the risk-free interest rate.

In contrast, the risk premium is reduced. The increase in climate damages decreases the correlation between stock market returns and consumption. As a result, the risk of holding a risky asset diminishes during recessions, leading to a lower premium.

On the environmental side, the marginal damages of carbon increase, calling for a higher SCC when the carbon lifetime is extended.

Finally, regarding the robustness of the main asset pricing effect, the integration of macro-finance with the carbon tax remains relatively stable, regardless of the carbon decay rate.

D Additional Results from Bayesian Estimation

D.1 Assessing Long-Run Growth Consistency with Empirical Evidence

To evaluate whether our model generates long-run growth consistent with empirical evidence, we utilize the Christiano-Fitzgerald bandpass filter to separate the long-run component from the business cycle component. Specifically, we filter out fluctuations lying between 2 and 32 quarters from the consumption data. The remaining component is interpreted as the medium/long-term trend of the data. This section is based on the work of Kung and Schmid (2015) that is aimed to include a medium term component into an otherwise asset pricing model, recognized in this literature as long run risk.

Figure 7 presents the filtered data. The blue line represents the low-frequency component of the data (with the left axis), while the orange line (with the right axis) depicts γ_t, the long-term growth component generated by our endogenous growth model. Although these series do not correspond exactly in their definitions, comparing them allows us to assess their relationship.

From the figure, we observe that both series exhibit similar magnitudes in their fluctuations. The correlation between the two series is also high at 40%, indicating significant comovement. Quantitatively, the size of their fluctuations is quite similar, suggesting that the long-term growth generated by our model aligns well with the empirical long-run trend.

Overall, the analysis indicates that the model's endogenous growth mechanism is capable of capturing long-term growth trends observed in the data, supporting the robustness of the model in reflecting empirical long-term economic dynamics.

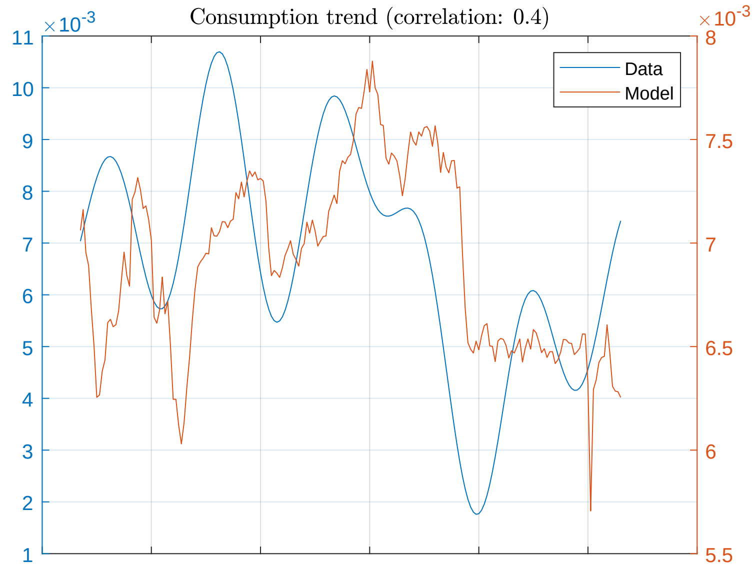
D.2 Time Path of the Carbon Stock

This section discusses the path of the carbon tax in the Bayesian estimation. Similar paths can be obtained using SMM as the methodologies are similar.

Figure 8 shows the time path of the carbon stock from before the sample starts in 1973 (first dot in green or red lines) to the end of the simulations (second dot in green/red lines). All our simulations and analyses rely on sampling between these two dots.

Figure 7: Comparison of Long-Run Consumption Trends between Model and Data

This figure compares the low-frequency component of consumption data, extracted using the Christiano-Fitzgerald bandpass filter (blue line with left axis), with the long-term growth component generated by the model, γ (orange line with right axis). The blue line represents the empirical trend, while the orange line captures long run risk.



To accommodate perturbation methods with the trend effect related to the carbon stock, which are crucial in the integrated assessment literature, we use perturbation methods in the long-run economy (at steady state). However, we initialize such that the anthropogenic carbon stock is about 120 Gt in 1973, consistent with the laissez-faire data.

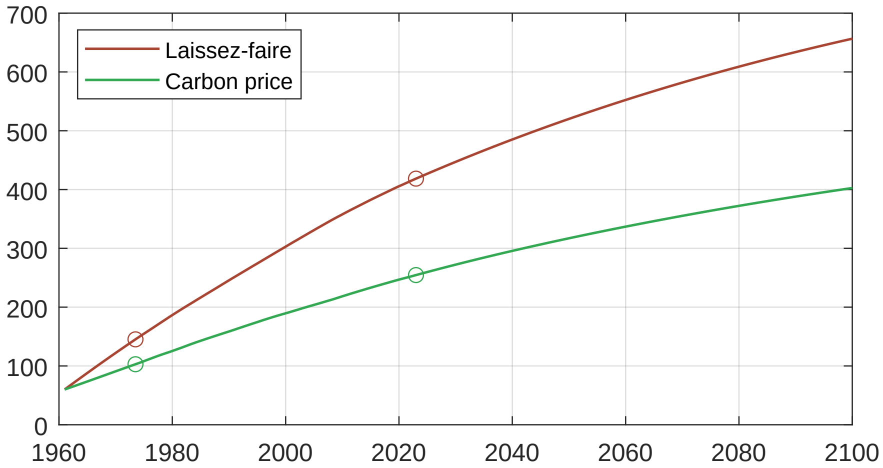
To obtain initial conditions in SMM/Bayesian estimation that are consistent with the distribution of shocks and the nonlinear effects on the unconditional mean of the variable, we pre-sample 200 parallel chains of size 50 periods and use the mean from the simulation as the initial condition (the first dot). Note that the stock of carbon is initialized to 60 Gt in 1960 as reported in Figure 8, which through our sampling method, yield 120 Gt of carbon in 1973 as mentioned in the paper.

Because the initial carbon stock is below its steady state, the stock of carbon will gradually rise, consistent with IPCC estimates, and provide a warming trajectory consistent with world data.

At the end of the sample period (the last dot), our framework, adjusted for risk (unlike IAM models), can be used to simulate and make out-of-sample projections. These projections can include any financial variables such as risk premia and stochastic discount

Figure 8: Time Path of Carbon Stock under Carbon Price and Laissez-Faire Policies

The figure reports the time path of the carbon stock (anthropogenic stock without pre-industrial stock) from 1960 to 2100 under carbon price and laissez-faire policies. The paths are simulated with estimated shocks using Bayesian techniques. Each dot delimits the time period of our sample, on which all our analysis is based.



rates.

D.3 Posterior Distributions for the Baseline Model

In this section, we analyze the prior versus posterior distributions of the parameters for the EZWH model. Figure 9 illustrates the comparison, with prior distributions depicted as dotted black lines and posterior distributions shown in blue.

Overall, the parameters are well identified, as evidenced by the clear divergence between the prior and posterior distributions. This divergence indicates that the data provided significant information to update the posterior distributions from the priors.

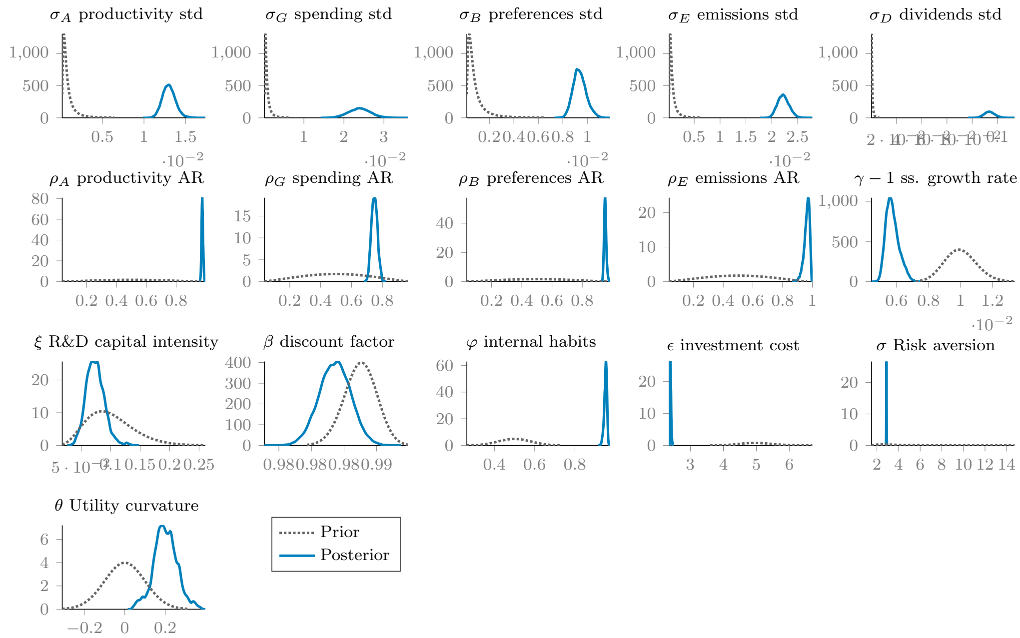
Some parameters are particularly well-informed by the data, as reflected by their narrow posterior distributions. These parameters exhibit high precision, suggesting that the model fit deteriorates substantially when these parameters deviate from their posterior means.

The parameter for the discount factor is an exception; while its posterior mean has shifted from the prior, its standard deviation has remained similar. This indicates that the data were less informative for this parameter compared to others.

The results demonstrate the model's capability to use empirical data effectively, thus reinforcing the reliability and robustness of the EZWH model in capturing the dynamics of the studied economic environment.

Figure 9: Prior vs. Posterior Distributions for EZWH Model Parameters

This figure compares the prior (dotted black lines) and posterior (blue lines) distributions of the EZWH model parameters. The data's informativeness is highlighted by the shift and narrowing of the posterior distributions compared to the priors.



D.4 Estimated Parameters under Alternative Preferences

Table IX reports the estimated parameters using Bayesian techniques. Note that all prior specifications are the same across different models, while the last line of the table reports the marginal data density, a measure of the relative fit of each model for a given sample. This is discussed further in the paper concerning the empirical dominance of habit models.

In terms of estimated shocks, each model provides very similar structures for autoregressive stochastic processes. Models with habits tend to exhibit relatively higher persistence of shocks, which can be explained by the consumption smoothing effects. The latter tends to make agents more sensitive to fluctuations in the marginal utility of consumption, attenuating in turn the macro volatility generated by this class of preferences. Therefore, it commands relatively higher persistence of shocks to generate sufficient volatility of consumption.

In terms of capital intensity in R&D, each model exhibits differences, with habits typically showing much lower values than recursive and power utility. Habits create endogenous persistence, and therefore require relatively less capital to create sufficient time variation in long-run risk. The growth, as well as the discount, are relatively different between CRRA and other versions. The CRRA accommodates low sensitivity to risk, and therefore a small precautionary motive, by a relatively high risk aversion coefficient. As our model includes growth, a high σ affects the discount factor, which would increase the steady-state interest rate. The model accommodates this by lowering the steady-state growth and increasing the discount factor for CRRA.

Table IX: Prior and Posterior distributions of structural parameters under alternative utility functions.

This table reports the means and the 5th and 95th percentiles of the posterior distributions drawn from eight parallel Markov chain Monte Carlo chains of 10,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hastings algorithm with a jump scale factor to match an average acceptance rate close to 25–30 percent for each chain. $\mathcal{B}$ denotes the Beta, $\mathcal{IG}_2$ the Inverse Gamma (type 2), $\mathcal{IG}$ the Inverse Gamma (type 1), $\mathcal{G}$ the Gamma.

	Prior	Posterior Distribution Mean [5%:95%]			
	Shape[Mean;Std]	EZWH	EZW	HABITS	CRRA
Panel A: Shock processes					
TFP $std(\eta_{At})$	$\mathcal{IG}_2$[0.001;0.0033]	0.013 *[0.012:0.014]*	0.011 *[0.011:0.012]*	0.012 *[0.011:0.013]*	0.013 *[0.012:0.014]*
Pref. $std(\eta_{Bt})$	$\mathcal{IG}_2$[0.001;0.0033]	0.009 *[0.009:0.010]*	0.016 *[0.01:0.018]*	0.009 *[0.008:0.010]*	0.013 *[0.012:0.014]*
Dividend $std(\eta_{Dt})$	$\mathcal{IG}_2$[0.001;0.0033]	0.094 *[0.090:0.101]*	0.087 *[0.081:0.090]*	0.098 *[0.091:0.106]*	0.090 *[0.088:0.093]*
Emission $std(\eta_{Et})$	$\mathcal{IG}_2$[0.001;0.0033]	0.022 *[0.020:0.024]*	0.023 *[0.020:0.025]*	0.023 *[0.021:0.024]*	0.022 *[0.020:0.024]*
Spending $std(\eta_{Gt})$	$\mathcal{IG}_2$[0.001;0.0033]	0.024 *[0.019:0.028]*	0.039 *[0.033:0.044]*	0.016 *[0.014:0.019]*	0.052 *[0.048:0.055]*
TFP AR ρ_A	$\mathcal{B}$[0.5;0.2]	0.975 *[0.967:0.984]*	0.983 *[0.975:0.989]*	0.993 *[0.993:0.990]*	0.957 *[0.951:0.964]*
Pref. AR ρ_C	$\mathcal{B}$[0.5;0.2]	0.952 *[0.940:0.965]*	0.669 *[0.576:0.727]*	0.939 *[0.926:0.952]*	0.730 *[0.710:0.749]*
Emissions AR ρ_E	$\mathcal{B}$[0.5;0.2]	0.963 *[0.927:0.987]*	0.974 *[0.956:0.989]*	0.987 *[0.980:0.993]*	0.893 *[0.885:0.900]*
Sprending AR ρ_G	$\mathcal{B}$[0.5;0.2]	0.748 *[0.717:0.782]*	0.890 *[0.848:0.922]*	0.700 *[0.679:0.722]*	0.869 *[0.849:0.889]*
Panel B: Structural parameters					
Capital in R&D ξ	$\mathcal{B}$[0.1;0.04]	0.074 *[0.050:0.100]*	0.151 *[0.118:0.193]*	0.042 *[0.026:0.057]*	0.161 *[0.137:0.186]*
Ss growth g	$\mathcal{G}$[0.01;0.001]	0.006 *[0.005:0.007]*	0.006 *[0.005:0.004]*	0.007 *[0.007:0.008]*	0.004 *[0.004:0.005]*
Discount factor β	$\mathcal{B}$[0.985;0.001]	0.983 *[0.982:0.985]*	0.987 *[0.985:0.989]*	0.984 *[0.983:0.986]*	0.988 *[0.987:0.989]*
Investment cost ϵ	$\mathcal{G}$[5;0.5]	2.405 *[2.382:2.436]*	3.967 *[3.602:4.412]*	2.560 *[2.542:2.576]*	4.231 *[4.214:4.249]*
Capital depr. δ_K	$\mathcal{B}$[0.01;0.005]	0.010 *[0.008:0.013]*	0.013 *[0.006:0.018]*	0.015 *[0.013:0.018]*	0.037 *[0.033:0.040]*
Habit smoothing ϖ	$\mathcal{B}$[0.5;0.08]	0.952 *[0.939:0.962]*	-	0.906 *[0.892:0.917]*	-
Utility curvature θ	$\mathcal{N}$[0;0.1]	0.201 *[0.104:0.297]*	-0.334 *[-0.373:-0.295]*	-	-
Risk aversion σ	$\mathcal{IG}$[4;3]	2.884 *[2.857:2.913]*	92.46 *[88.354:95.756]*	1.739 *[1.701:1.774]*	3.473 *[3.458:3.488]*
Log marginal data density		-1 788.27	-1 824.62	-1 789.88	-1 847.20

Regarding investment curvature, models with habits generate relatively more consumption smoothing, which lowers macroeconomic volatility. They require a lower investment cost to capture fluctuations in investment, while competing formulations require a higher parameter to sufficiently reduce the volatility of investment. Regarding capital depreciation, the CRRA provides a relatively large capital depreciation.

Regarding the degree of slow-moving habits, the habit model alone requires a lower coefficient to generate a stronger degree to match risk premia and the risk-free rate. Indeed, a smaller degree of habits implies increased sensitivity to consumption fluctuations and amplifies risk aversion during economic downturns, leading to higher demand for risk compensation and precautionary savings. Because EZWH comprises two complementary mechanisms to generate sensitivity to risk, it requires relatively less consumption smoothing (from consumption habits) and less disutility from uncertainty (from recursive preferences). In contrast, EZW requires a much larger risk aversion coefficient, as in Rudebusch and Swanson (2012) and Basu and Bundick (2017). This typically occurs in perturbation methods, unlike projection methods, as perturbations provide an approximation up to a certain order, neglecting in turn all other nonlinearity of larger orders.

D.5 Endogenous Uncertainty and Carbon Pricing under a CRRA Utility Function

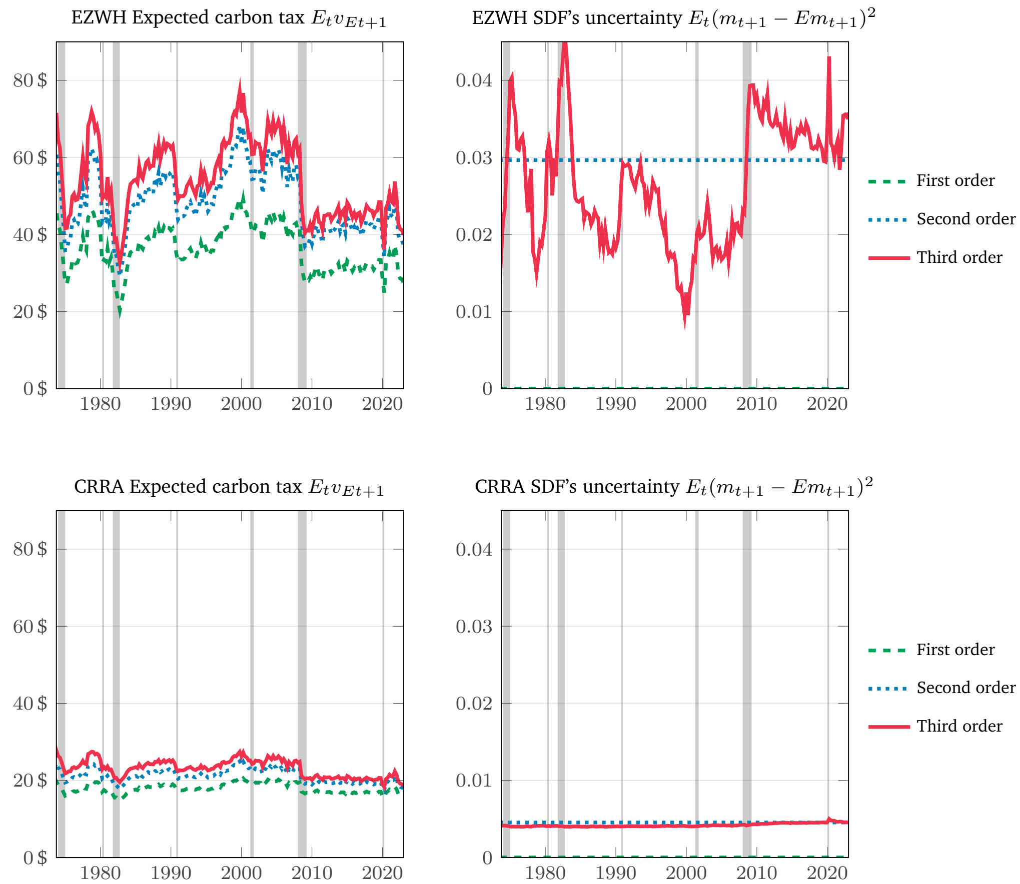
The provided figure compares the expected carbon tax and the stochastic discount factor (SDF) uncertainty for two models: EZWH (Epstein-Zin preferences with habit formation) and CARA (Constant Absolute Risk Aversion) preferences, across different orders of approximation (first, second, and third). This comparison highlights the impact of higher-order terms and endogenous uncertainty on the carbon tax.

The left panels illustrate the expected carbon tax ($E_t \tau_{Et+1}$) over time, while the right panels depict the SDF's uncertainty ($E_t(m_{t+1} - Em_{t+1})^2$) over the same period. Different lines represent different orders of approximation (first, second, and third). In the EZWH model (top panels), the expected carbon tax is highly pro-cyclical, especially at higher-order approximations. This is evidenced by the significant increase in the carbon tax during economic expansions (e.g., the tech bubble in the 2000s) and its sharp decline during recessions (e.g., the 2008 financial crisis and the COVID-19 pandemic). In contrast, the CARA model (bottom panels) shows relatively stable carbon tax levels with small fluctuations across all orders of approximation, indicating less sensitivity to uncertainty.

At the first-order approximation, both models show some level of pro-cyclicality, but the effect is minimal. The second-order approximation introduces the variance of the state variables ($\text{var}_t(z_t)$ and $h_{\sigma\sigma}\sigma^2$), causing a permanent increase in the carbon tax due to the precautionary saving motive and prudence effect. The third-order approximation captures skewness and other higher-order moments, leading to substantial time-varying risk premia. In the EZWH model, this results in a more pronounced pro-cyclicality of the carbon tax. The right panels highlight the endogenous uncertainty of the SDF. In the EZWH model, higher-order approximations significantly increase the time variation of SDF's uncertainty, aligning with the notion that fluctuations in uncertainty are critical

Figure 10: The Role of Uncertainty in Shaping the Price of Carbon

The gray shaded areas are NBER-dated recessions in the U.S. Each simulation represents the path of the variable with the model approximated up to a specific order of the Taylor expansion. To obtain the path under power utility on the left panel, IES/RA separation & habits formation are disabled.



for capturing financial data features. For the CARA model, SDF's uncertainty remains relatively constant, reflecting the lack of endogenous risk introduced by higher-order terms.